

Trig 4.1

Determine roots of polynomial equations *

Apply the fundamental theorem of algebra (# of roots) *

factor ... x-factor

polynomial (in one variable),

degree **highest exp.**

leading coefficient

polynomial function

zeros (real)

polynomial equation

Roots (can be real or imag.)

imaginary number

real number

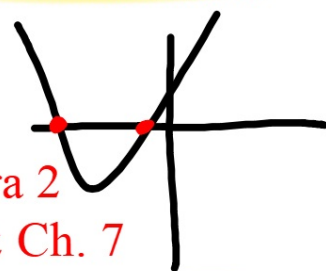
complex number

Fundamental Theorem of Algebra

degree = # answers (at most)

$$x^2 + 5x + 6 \quad \cancel{2} \times \cancel{2} \quad (x+3)(x+2)$$

$1000x^{18} + 500x^{10} + 250x^5$ is a polynomial in one variable.



$$y = f(x) = x^2 + 5x + 6$$

*Algebra 2
Ch. 5 & Ch. 7

$$0 = x^2 + 5x + 6$$

$x = -3$
 $x = -2$
(or 5.5)

$$\sqrt{25} = 5$$

$$5^2 = 25$$

$$\sqrt{-25} = \sqrt{25} \cdot \sqrt{-1}$$

$5i$

$$i = \sqrt{-1}$$

$$i^2 = i \cdot i = \sqrt{-1} \cdot \sqrt{-1} = -1$$

$$i^3 = (i \cdot i) \cdot i = \underbrace{\sqrt{-1} \cdot \sqrt{-1}}_{-1} \cdot \underbrace{(\sqrt{-1})}_{i} = -i$$

$$i^4 = \boxed{i \cdot i \cdot i \cdot i} = \underbrace{\sqrt{-1} \cdot \sqrt{-1}}_{-1} \cdot \underbrace{\sqrt{-1} \cdot \sqrt{-1}}_{-1} = 1$$

$$i^{35} = i^{32} \cdot i^3 = (i^4)^8 \cdot i^3 = 1^8 \cdot (-i) = -i$$

2 Consider the polynomial function $f(x) = x^3 - 6x^2 + 10x - 8$.

a. State the degree and leading coefficient of the polynomial.

b. Determine whether 4 is a zero of $f(x)$. *yes*

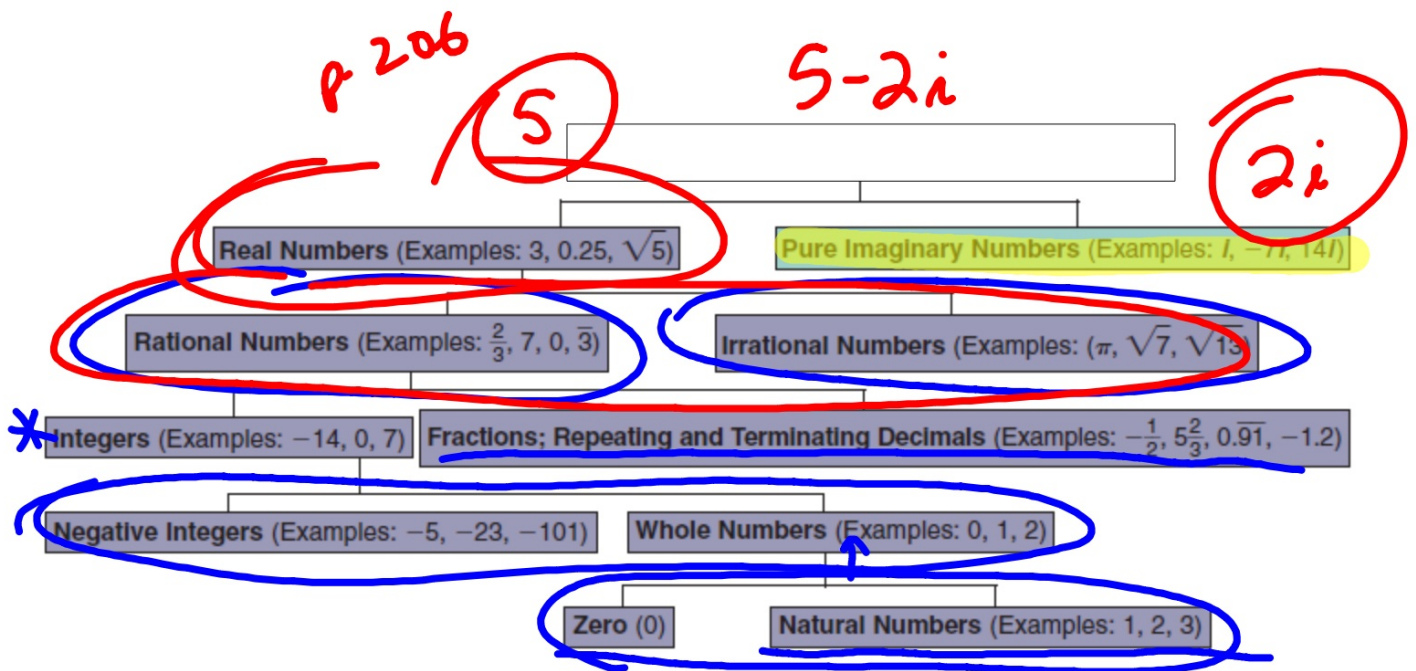
$$\begin{aligned} 0 & \stackrel{?}{=} x^3 - 6x^2 + 10x - 8 \\ & 4^3 - 6 \cdot 16 + 40 \\ & = 64 - 96 + 40 - 8 \end{aligned}$$

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$5-2i$

5

$2i$



Q.207

Fundamental Theorem of Algebra	Every polynomial equation with degree greater than zero has at least one root in the set of complex numbers.
A corollary to the Fundamental Theorem of Algebra states that the degree of a polynomial indicates the number of possible roots of a polynomial equation.	
Corollary to the Fundamental Theorem of Algebra	Every polynomial $P(x)$ of degree n ($n > 0$) can be written as the product of a constant k ($k \neq 0$) and n linear factors. $P(x) = k(x - r_1)(x - r_2)(x - r_3) \dots (x - r_n)$ Thus, a polynomial equation of degree n has exactly n complex roots, namely $r_1, r_2, r_3, \dots, r_n$.

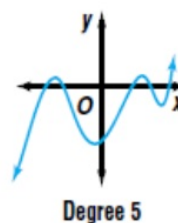
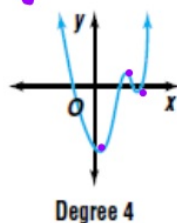
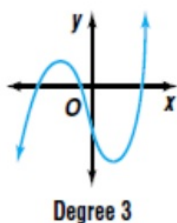
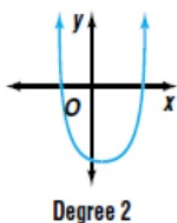
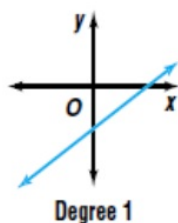
Degree = number of possible roots

(...could be double root $x=3$ or $x=3$)
(might not be real)

Complex conjugate partners (alg 2)

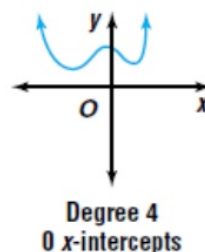
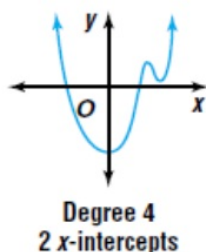
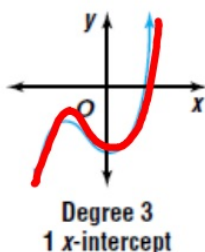
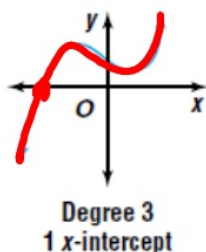
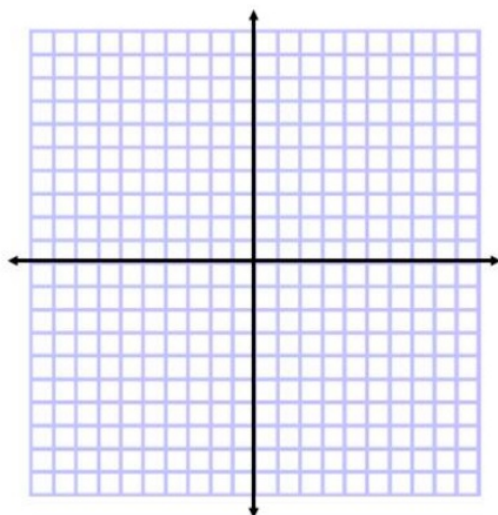
N roots: At most... (some could also be complex conjugate partner pairs)

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degree =
zeros
roots
(at most)

deg-1 = t.p.
(at most)



~~6~~
~~3~~

Solve the equation:

$$x^2 + 3x + 6 = 0$$

$$x = \frac{-3 \pm \sqrt{9 - 4 \cdot 1 \cdot 6}}{2}$$

$$= \frac{-3 \pm \sqrt{9 - 24}}{2}$$

$$= \frac{-3 \pm \sqrt{-15}}{2}$$

real imag

$$= \frac{-3 \pm i\sqrt{15}}{2}$$

$$\frac{-3}{2} \pm \frac{i\sqrt{15}}{2}$$

↑

↑

Write the equation with roots $x = 2$ and $x = -5$
How many factors? Work backwards!

$$x^2 + 3x - 10 = 0$$

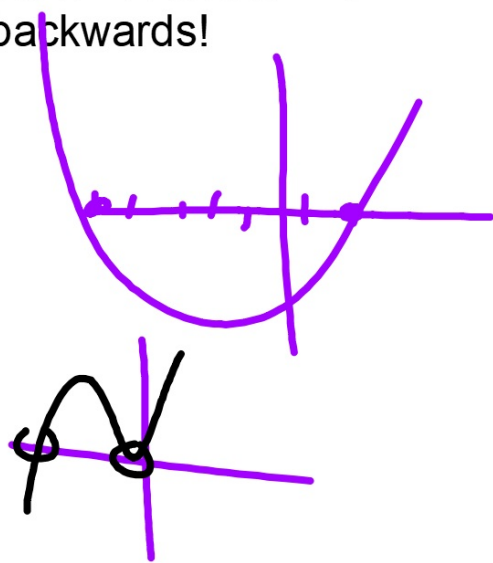
$$(x-2)(x+5) = 0$$

$$x-2=0$$

$$x+5=0$$

$$x=2$$

$$x=-5$$



Complex conjugate partners...

$$2 + 5i \quad 2 - 5i$$

$$x^3 - 2x^2 + 16x - 32 = 0$$

- 3 a. Write a polynomial equation of least degree with roots 2, $4i$, and $-4i$.
 b. Does the equation have an odd or even degree? How many times does the graph of the related function cross the x-axis?

$$(x-2)(x^2+16)=0$$

$$(x-2)(x-4i)(x+4i)=0$$

$$x-2=0$$

$$x=2$$

$$-2 \quad -2$$

$$x-4i=0$$

$$x=4i$$

$$-4i \quad -4i$$

$$x+4i=0$$

$$x=-4i$$

$$+4i \quad +4i$$

Did they tell you ALL the roots? They don't have to...

$$\begin{array}{r} x-4i \quad -1 \\ x+4i \quad \uparrow \\ \hline 4ix - 16i^2 \\ -4ix \end{array}$$

$$\begin{array}{r} x^2 + 16 \\ x-2 \\ \hline x^3 - 2x^2 - 32 \end{array}$$

$$x^3 \quad 16x$$

$$y = x^2 - 1$$

- 4 State the number of complex roots of the equation $x^2 - 1 = 0$. Then find the roots and graph the related function.

factor...

$$(x+1)(x-1) = 0$$

$$\downarrow \quad \downarrow$$

$$x+1=0 \quad x-1=0$$

$$x=-1 \quad x=1$$

