Trig 4.5

Approximate real zeros of a polynomial function
Use the upper and lower bound theorems

zero (of a function)

rational

real

Descarte's rule of signs

location principle

upper bound

lower bound

Use technology to help! But you still have to prove it!

$f(x) = x^3 - 3x^2 + x - 1$

UB 3 | 1 | -3 | 1 | -1
↓ | | 2 |
1 | -1 |

UB $x = 3$

UB $= 3 = x$

$f(-x) = -x^3 - 3x^2 - x - 1$

LB -1 | -1 | -3 | -1 | -1
↓ | | 1 | 2
-1 | -2 | 1

LB $x = 0$

LB $= 0 = x$

$f(x) = x^3 - 2x^2 - 4x + 5$

$-2 < x < -1$ ☺

$x = 1$ ☺

~~$3 < x < 4$~~

$2 < x < 3$ ☺

$f(x) = x^3 - 2x^2 - 4x + 5$ $f(-x) = -x^3 - 2x^2 + 4x + 5$

UB

3 | 1 | -2 | -4 | 5

↓ | | 3 | 3 |

| 1 | 1 | -1 |

UB = 4 **UB x=4**

LB

2 | -1 | -2 | 4 | 5

↓ | | -2 | -8 | -8 |

| -1 | -4 | -4 | -3 |

LB -2 (?) **LB = -2**

graph...table

Approximate the real zeros of each function to the nearest tenth.

19. $f(x) = 3x^4 + x^2 - 1$

20. $f(x) = x^2 + 3x + 1$

Upper bound: There is only end behavior to the right of ...
(no max, min, zeros, etc.)
f(x) ...no sign changes in depressed polynomial

Upper Bound Theorem	Suppose c is a positive real number and $P(x)$ is divided by $x - c$. If the resulting quotient and remainder have no change in sign, then $P(x)$ has no real zero greater than c. Thus, c is an upper bound of the zeros of $P(x)$.

Zero coefficients are ignored when counting sign changes.

Lower bound: There is only end behavior to the left of...
(no max, min, zeros, etc.)
f(-c) ...no sign changes in depressed polynomial then c is a lower bound
(you try the opposite)
Goal: to zoom in as tightly as possible

A **lower bound** is an integer less than or equal to the least real zero. A lower bound of the zeros of $P(x)$ can be found by determining an upper bound for the zeros of $P(-x)$.

Lower Bound Theorem	If c is an upper bound of the zeros of $P(-x)$, then $-c$ is a lower bound of the zeros of $P(x)$.

Use the Upper Bound Theorem to find an integral upper bound and the Lower Bound Theorem to find an integral lower bound of the zeros of each function.

26. $f(x) = 3x^3 - 2x^2 + 5x - 1$

27. $f(x) = x^2 - x - 1$

$f(x) = x^2 - x - 1$

$f(-x) = x^2 + x - 1$

1 | 1 -1 -1
1 0 -1

UB = 2

0 | 1 1 -1
1 1 -1

LB = -1

29. $f(x) = x^3 + 5x^2 - 3x - 20$

$-x^3 + 5x^2 + 3x - 20$

UB

1	1	5	-3	-20
	↓	1	6	3
	1	6	3	

UB = 2

LB

-1	-1	5	3	-20
	↓	-6	-6	-18
	-1	-1	-3	

LB = -6

$$f(x) = x^5 + 5x^4 - 3x^3 + 20x^2 - 15$$

$$-x^5 + 5x^4 + 3x^3 + 20x^2 - 15$$

UB of

1	5	-7
↓	0	?
1	5	-

$UB = 1$

6

-1	5	3	20	0	-15
↓	-6	-6	-18		
-1	-1	-3	?		

$LB = -7$

-6

WB 4.5
1-14