

Trig 4.1

Determine roots of polynomial equations *

Apply the fundamental theorem of algebra (# of roots) *

degree

leading coefficient

polynomial function
zeros (real)

$$f(x) = \text{~~~~~}$$

*Algebra 2 Ch. 5
& Ch. 7

polynomial equation

Roots (can be real or imag.)

$$\text{~~~~~} = 0$$

imaginary number

$$2 + 5i \quad 2 - 5i$$

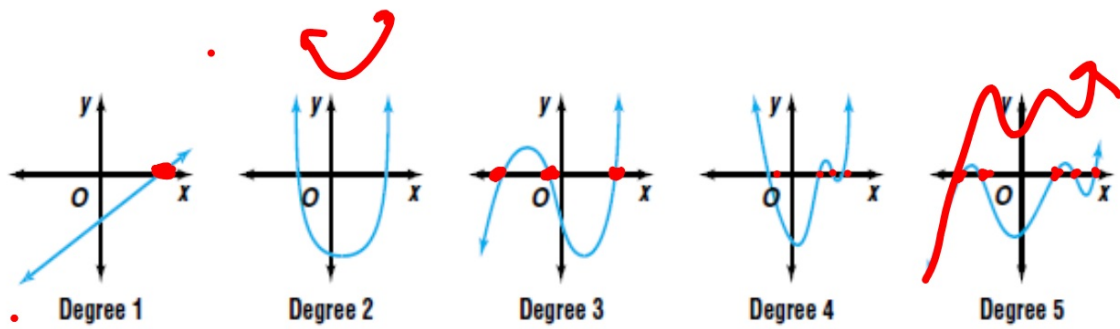
real number

complex number

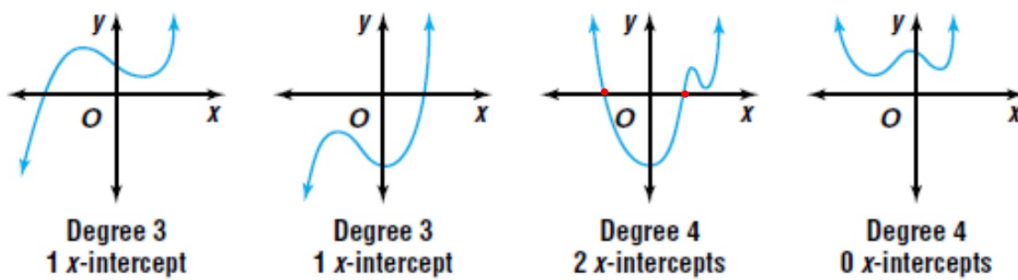
$$x = -b \pm \sqrt{\quad}$$

Fundamental Theorem of Algebra

$$\text{roots} = \text{degree} \\ (\text{at most})$$



FTA: equation of degree n has **at most** n real solutions



State the degree and leading coefficient of each polynomial.

5. $a^3 + 6a + 14$

6. $5m^2 + 8m^5 - 2$

$$5x'y^2$$

Determine whether each number is a root of $x^3 - 5x^2 - 3x - 18 = 0$. Explain.

7. 5

8. 6

Write a polynomial equation of least degree for each set of roots. Does the equation have an odd or even degree? How many times does the graph of the related function cross the x-axis?

Alg 2 4.3

9. -5, 7

10. 6, $2i$, $-2i$, i , $-i$

= 0

$$x^2 - 2x - 35 = 0$$

$$(x+5)(x-7) = 0$$

$$x+5=0 \quad x-7=0$$

$$x = -5 \quad x = 7$$

conjugate pair

$$x^5 - 6x^4 + 5x^3 - 3x^2 + 4x - 24 = 0$$

$$(x-6)(x^4 + 5x^2 + 4) = 0$$

$$(x-6)(x^2+4)(x^2+1) = 0$$

$$(x-6)(x-2i)(x+2i)(x-i)(x+i) = 0$$

$$x-6=0$$

$$x-2i=0$$

$$x+2i=0$$

$$x-i=0$$

$$x+i=0$$

$$x=6$$

$$x=2i$$

$$x=-2i$$

$$x=i$$

$$x=-i$$

$$\begin{array}{r} x^4 + 5x^2 + 4 \\ x - 6 \end{array}$$

$$\begin{array}{r} -6x^4 - 30x^2 - 24 \\ x^5 \quad 5x^3 + 4x \end{array}$$

$$x^3 = 0$$

$$x=3 \quad x=2-i$$

$$(x-3)(x^2-4x+5)=0$$

$$(x-3)(x-2+i)(x-2-i)=0$$

$$x-3=0$$

$$x=3$$

$$x-2+i=0$$

$$x=2-i$$

$$x-2-i=0$$

$$x=2+i$$

$$\begin{array}{r} x-2+i \\ x-2-i \\ \hline (-ix)(2i) - i \cdot i \\ -2x+4 \quad 2x \\ \hline x^2-2x+4 \end{array}$$

State the number of complex roots of each equation. Then find the roots and graph the related functions.

11. $x^2 - 14x + 49 = 0$

12. $a^3 + 2a^2 - 8a = 0$

13. $t^4 - 1 = 0$

$f(x) = x^2 - 14x + 49$

$(x-7)(x-7)$

$x=7 \quad x=7$

$a(a^2 + 2a - 8) = 0$

$a(a-2)(a+4) = 0$

Use everything that you know:

of roots 3

factoring

crossing points

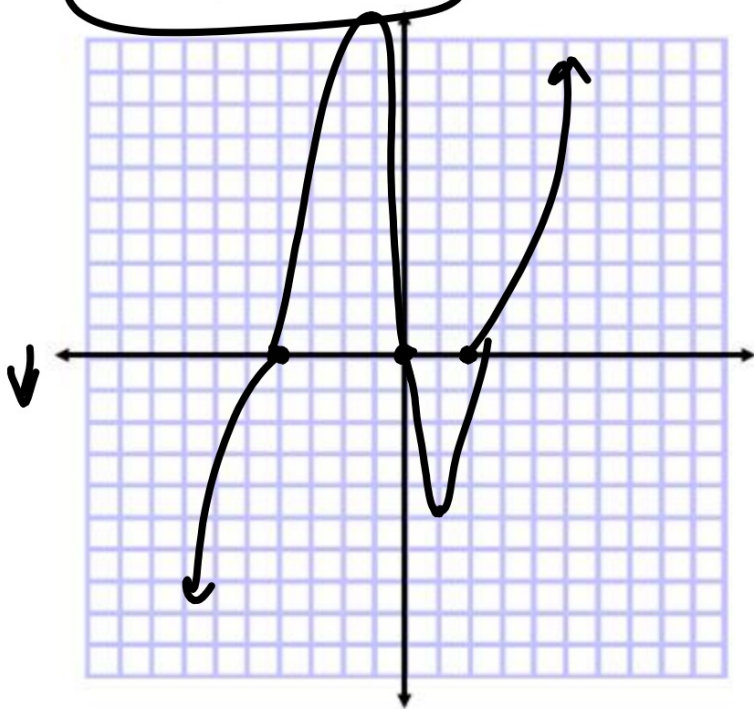
end behavior

$-8 + 8 + 16$

Show a (small) table of values

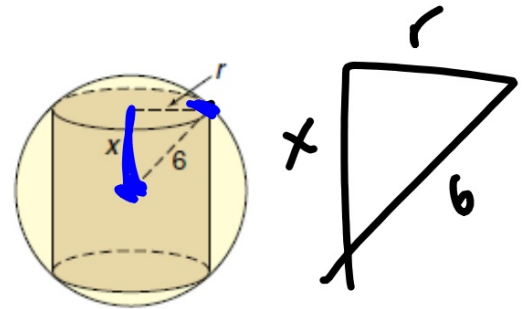
$$\begin{array}{r|l} 8 & 64 - 112 + 49 \\ 9 & 81 - 126 + 49 \end{array}$$

$$\begin{array}{r|l} & 1 + 2 \rightarrow \\ -2 & \end{array}$$



14. **Geometry** A cylinder is inscribed in a sphere with a radius of 6 units as shown.

- Write a function that models the volume of the cylinder in terms of x . (Hint: The volume of a cylinder equals $\pi r^2 h$.)
- Write this function as a polynomial function.
- Find the volume of the cylinder if $x = 4$.



$$V = \pi r^2 h$$

$$V = \pi (\sqrt{36-x^2})^2 (2x)$$

$$V = \pi (36-x^2)(2x)$$

$$f(x) = \pi (72x - 2x^3)$$

$$= \pi (72 \cdot 4 - 2 \cdot 4^3) = \pi (288 - 128) = 160\pi$$

$$\begin{aligned} x^2 + r^2 &= 36 \\ \sqrt{r^2} &= \sqrt{36-x^2} \\ r &= \sqrt{36-x^2} \end{aligned}$$

It's messy, don't be scared.
Exact answer (in terms of π)

4.1 WB