Precalc 10.7

Find the equations of conic sections that have been rotated or translated
Graph rotations and/or translations of conics
Use the discriminant to identify conic sections
Find the angle of rotation for a given equation

Quiz 10.7-10.8 Mon.

rotation

Bxy

translation

discriminant

degenerate case

2) $-(x+2)^2 + 2(x+2)(y-6) - 1(y-6)^2 + 6(y-6) = 4$

$-(x^2+4x+4) + 2(xy-6x+2y-12) - (y^2-12y+36) + 6(y-6) = 4$

$3^2 = 9$

$-x^2-4x-4+2xy-12x+4y-24-y^2+12y-36+6y-36 = 4$

$-1x \cdot -1x = x^2$

$-x^2-y^2+2xy-16x+22y-104 = 0$

$5(-x)^2$

$-x \cdot -x$

$2S\ x^2$

$-x \cdot -x$

$5(-1x)^2 = 5 \cdot -1x \cdot -1x$

3) $\theta = 90$

$(x\cos\theta + y\sin\theta)^2 + 4(x\cos\theta + y\sin\theta)(-x\sin\theta + y\cos\theta) + 5(-x\sin\theta + y\cos\theta)^2 - 7(x\cos\theta + y\sin\theta) - (-x\sin\theta + y\cos\theta) = 0$

$(y)^2 + 4(y)(-x) + 5(-x)^2 - 7(y) - (-x) = 0$

$-7(x\cos\theta + y\sin\theta) - (-x\sin\theta + y\cos\theta) = 0$

$a \cdot a = a^2$

$x \cdot x = x^2$

$-1x \cdot -1x = x^2$ (2)

$(x)^2$

$x \cdot x$

x^2

$-a \cdot -a$

a^2

$(-a)^2$

a^2

$-a^2$

$T_{(h, k)}$ ↔ translation with respect to (h, k)

replace x with (x-h)
replace y with (y-k)
(just like moving center of circles...)

To find the equation of a conic section with respect to a rotation of θ, replace

$$x \text{ with } x' \cos\theta + y' \sin\theta$$
$$\text{and } y \text{ with } -x' \sin\theta + y' \cos\theta.$$

Will write on board for test/quiz *but you have to know what to do with it…*

Identifying Conics By Using the Discriminant

For the general equation $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$,

- if $B^2 - 4AC < 0$, the graph is a circle ($A = C$, $B = 0$) or an ellipse ($A \neq C$ or $B \neq 0$);
- if $B^2 - 4AC > 0$, the graph is a hyperbola;
- if $B^2 - 4AC = 0$, the graph is a parabola.

Needed if something has been rotated Bxy...
If B=0 can still identify by inspection

$Ax^2+Bxy+Cy^2+Dx+Ey+F=0$

Angle of Rotation About the Origin

For the general equation $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$, the angle of rotation θ about the origin can be found by

$$\theta = \frac{\pi}{4}, \text{ if } A = C, \text{ or}$$

$$\tan 2\theta = \frac{B}{A - C}, \text{ if } A \neq C.$$

$\tan 2\theta = \frac{B}{A-C}$ $\quad$ $\theta = \frac{\pi}{4} = 45°$

$A=2 \quad B=9 \quad C=14$

4 Identify the graph of the equation $2x^2 + 9xy + 14y^2 - 5 = 0$. Then find θ and use a graphing calculator to draw the graph.

$$\tan 2\theta = \frac{9}{2-14}$$

$$= \frac{9}{-12}$$

$$\tan(2\theta) = -0.75$$

$$2\theta = -36.9^\circ$$

$$\theta = -18.4^\circ$$

tan2x = B/(A-C)
Desmos

$2(\quad)^2 + 9(\quad)(\quad) +$

$14(\quad)^2 - 5 = 0$

WB 10.7