

Precalc 10.7

Find the equations of conic sections that have been rotated or translated

Graph rotations and/or translations of conics

Use the discriminant to identify conic sections

Find the angle of rotation for a given equation

rotation

Bxy

translation

discriminant

degenerate case

$$B^2 - 4AC$$

Quiz 10.5-10.6

$T_{(h,k)}$ $\leftrightarrow$ translation with respect to (h,k)

replace x with $(x-h)$
replace y with $(y-k)$

(just like moving center of circles...)

$$x^2 + y^2 = 9$$

$$(x-3)^2 + (y+5)^2 = 9$$

Identify the graph of each equation. Write an equation of the translated or rotated graph in general form.

5. $x^2 + y^2 = 7$ for $T_{(3,2)}$

6. $y = 2x^2 - 7x + 5$ for $T_{(-4,5)}$

↑↑

★

$$y - 5 = 2(x + 4)^2 - 7(x + 4) + 5$$

★ $(x - 3)^2 + (y - 2)^2 = 7$

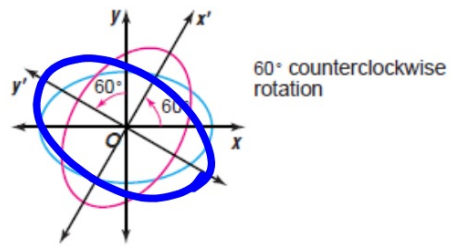
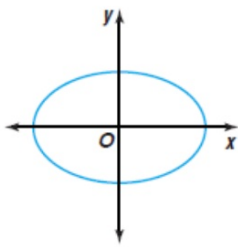
$$\cancel{x^2} - \cancel{6x} + 9 + \cancel{y^2} - \cancel{4y} + 4 - 7$$

$$x^2 + y^2 - 6x - 4y + 6 = 0$$

$$y - 5 = 2(x^2 + 8x + 16)$$

$$-y + 5 = 2x^2 + 16x + 32 - 7x - 28 + \cancel{5 - y} + 5$$

$$2x^2 + 9x - y + 14 = 0$$



default CCW

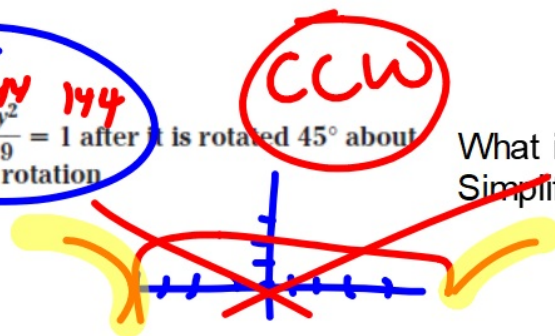
To find the equation of a conic section with respect to a rotation of θ ,
replace

$$x \text{ with } x' \cos \theta + y' \sin \theta$$
$$\text{and } y \text{ with } -x' \sin \theta + y' \cos \theta.$$

Will write on board for test/quiz *but you have to know what to do with it...*

- 2 Find the equation of the graph of $\frac{x^2}{16} - \frac{y^2}{9} = 1$ after it is rotated 45° about the origin. Then sketch the graph and its rotation.

What is it?
Simplify fractions first



$$9x^2 - 16y^2 = 144$$

x with $x' \cos \theta + y' \sin \theta$
and y with $-x' \sin \theta + y' \cos \theta$.

$$9(x' \cos 45 + y' \sin 45)^2 - 16(-x' \sin 45 + y' \cos 45)^2 = 144$$

$$9\left(\frac{\sqrt{2}}{2}x' + \frac{\sqrt{2}}{2}y'\right)^2 - 16\left(-\frac{\sqrt{2}}{2}x' + \frac{\sqrt{2}}{2}y'\right)^2 = 144$$

Does it still look like the equation of a hyperbola?

$$9\left(\frac{1}{2}(x')^2 + x'y' + \frac{1}{2}(y')^2\right) - 16\left(\frac{1}{2}(x')^2 - x'y' + \frac{1}{2}(y')^2\right) = 144$$

$$\frac{9}{2}(x')^2 + 9x'y' + \frac{9}{2}(y')^2 - 8(x')^2 + 16x'y' - 8(y')^2 = 144$$

$$2(-3.5(x')^2 + 25x'y' - 3.5(y')^2) = 144$$

$$-7(x')^2 + 50x'y' - 7(y')^2 = 288$$

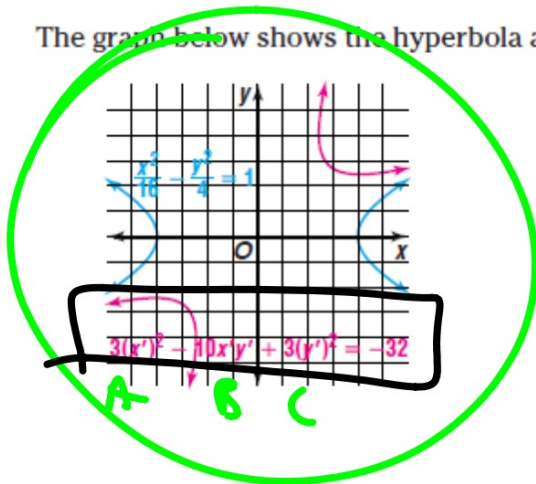
$$7(x')^2 - 50x'y' + 7(y')^2 = -288$$

$$7(x')^2 - 50x'y' + 7(y')^2 + 288 = 0$$

A B C

$$2500 - 196 = 2304$$

The graph below shows the hyperbola and its rotation.



$$100 - 4 \cdot 3 \cdot 3$$

$$100 - 36$$

$$64$$

Discriminant (alg 1 & 2)

Identifying
Conics By
Using the
Discriminant

For the general equation $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$,

- if $B^2 - 4AC < 0$, the graph is a circle ($A = C, B = 0$) or an ellipse ($A \neq C$ or $B \neq 0$);
- if $B^2 - 4AC > 0$, the graph is a hyperbola;
- if $B^2 - 4AC = 0$, the graph is a parabola.

Needed if something has been rotated Bxy...
If B=0 can still identify by inspection

When XY term (rotation) all bets are off..
sine 45, cos 30 etc. changes everything
 $b^2 - 4ac$

3 Identify the graph of the equation $x^2 - 4xy + 4y^2 + 5\sqrt{5}y + 1 = 0$.

13-3/ord 10.7

Angle of
Rotation
About the
Origin

For the general equation $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$, the angle of rotation θ about the origin can be found by

$$\theta = \frac{\pi}{4}, \text{ if } A = C, \text{ or}$$
$$\tan 2\theta = \frac{B}{A - C}, \text{ if } A \neq C.$$

- 4 Identify the graph of the equation $2x^2 + 9xy + 14y^2 - 5 = 0$. Then find θ and use a graphing calculator to draw the graph.

$$\tan 2\theta = \frac{B}{A-C}$$

Desmos

Graphing calculator: must solve for $y =$
Use quadratic formula
not sure it is worth it..

Desmos?

4 Identify the graph of the equation $2x^2 + 9xy + 14y^2 - 5 = 0$. Then find θ
and use a graphing calculator to draw the graph.