

Precalc 10.6

Recognize conic sections by their equations

Find a rectangular equation for a curve defined parametrically

Find a parametric equation for a curve defined rectangularly

general conic equation

parametric

$$x = f(t)$$

rect.

$$x^2 + y^2 \dots$$

parametric equation

$$y = f(t)$$

$\sin^2 + \cos^2 = 1$

 (pythagorean identity)

Graphing calculator: parametric mode

General Equation for Conic Sections

The equation of a conic section can be written in the form

$$Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0,$$

where A , B , and C are not all zero.

Which parts are present? Which parts are missing?

General Form: $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$		
Conic Section	Standard Form of Equation	Variation of General Form of Conic Equations
circle	$(x - h)^2 + (y - k)^2 = r^2$	$A = C$
parabola	$(y - k)^2 = 4p(x - h)$ or $(x - h)^2 = 4p(y - k)$	Either A or C is zero.
ellipse	$\frac{(x - h)^2}{a^2} + \frac{(y - k)^2}{b^2} = 1$ or $\frac{(y - k)^2}{a^2} + \frac{(x - h)^2}{b^2} = 1$	A and C have the same sign and $A \neq C$.
hyperbola	$\frac{(x - h)^2}{a^2} - \frac{(y - k)^2}{b^2} = 1$ or $\frac{(y - k)^2}{a^2} - \frac{(x - h)^2}{b^2} = 1$	A and C have opposite signs.
	$xy = k$	$A = C = D = E = 0$

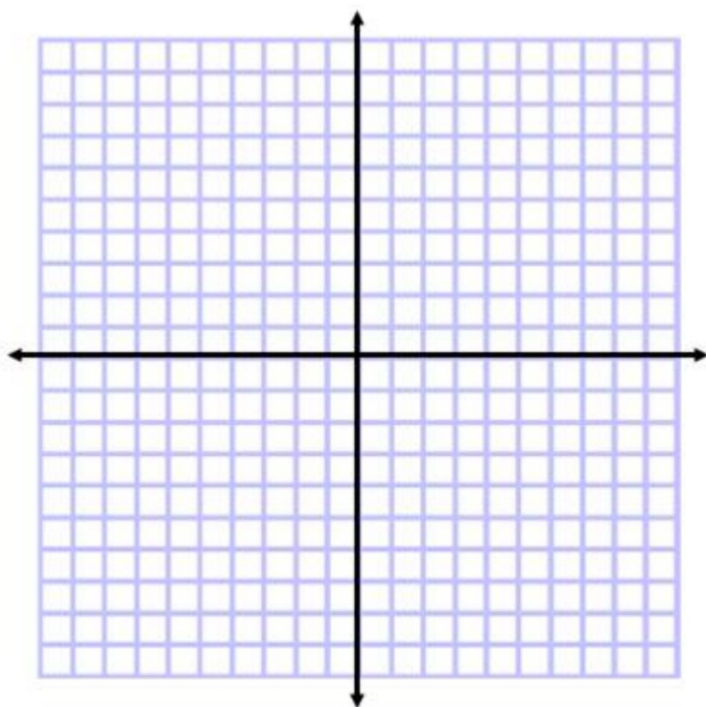
Remember that graphs can also be degenerate cases.

Works as long as there is no xy term (rotation)

- 2** Graph the curve defined by the parametric equations $x = 4t^2$ and $y = 3t$, where $-2 \leq t \leq 2$. Then identify the curve by finding the corresponding rectangular equation.

Table of values:

t	x	y
-2	$4(-2)^2$	$3(-2)$
-1		
0		
1		
2		



$$x^2 + y^2$$

$$\cos^2 x + \sin^2 x = 1$$

3 Find the rectangular equation of the curve whose parametric equations are $x = 2 \cos t$ and $y = 2 \sin t$, where $0 \leq t \leq 2\pi$. Then graph the equation using arrows to indicate how the graph is traced.

$$\cos^2 x + \sin^2 = 1$$

$$\left(\frac{1}{2}x\right)^2 + \left(\frac{1}{2}y\right)^2 = 1$$

$$\frac{1}{4}x^2 + \frac{1}{4}y^2 = 1$$

$$x^2 + y^2 = 4$$

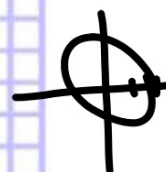
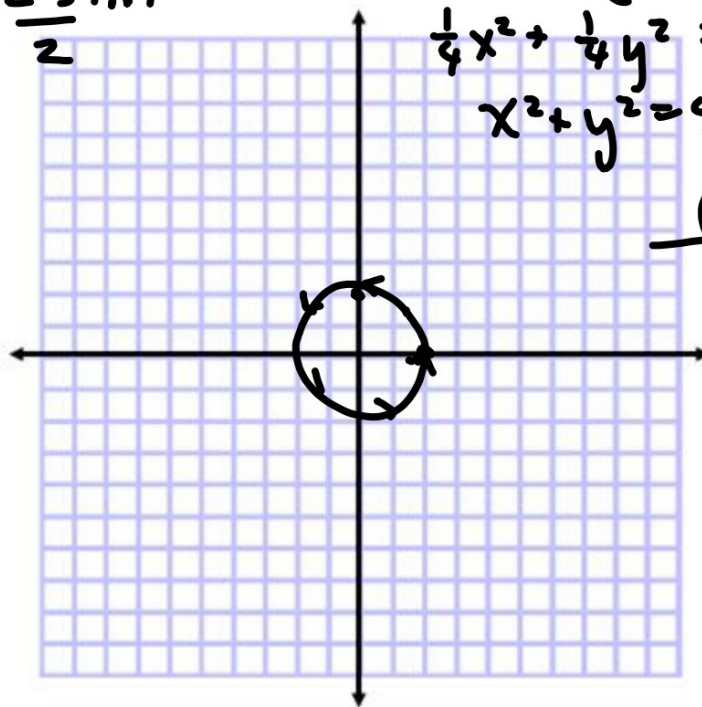


Table of values:

t	x	y
0	2	0
$\frac{\pi}{2}$	0	2
π	-2	0
$\frac{3\pi}{2}$	0	-2
2π	2	0

$$\frac{x}{2} = \frac{2 \cos t}{2}$$

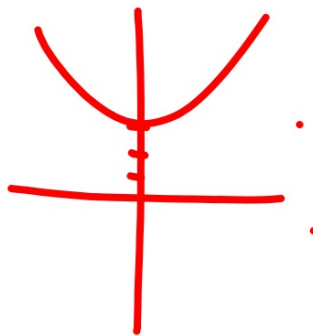
$$\frac{y}{2} = \frac{2 \sin t}{2}$$



4 Find parametric equations for the equation $y = x^2 + 3$.

let $x=t$
 $y=$

$$x = t$$
$$y = t^2 + 3$$



Find the rectangular equation of the curve whose parametric equations are given. Then graph the equation using arrows to indicate orientation.

8. $x = t, y = -t^2 - 6t + 2; -\infty < t < \infty$ 9. $x = 2 \cos t, y = 3 \sin t; 0 \leq t \leq 2\pi$

Table of values:

t x y

Table of values:

t x y

$$\cos T = \frac{1}{2}x$$

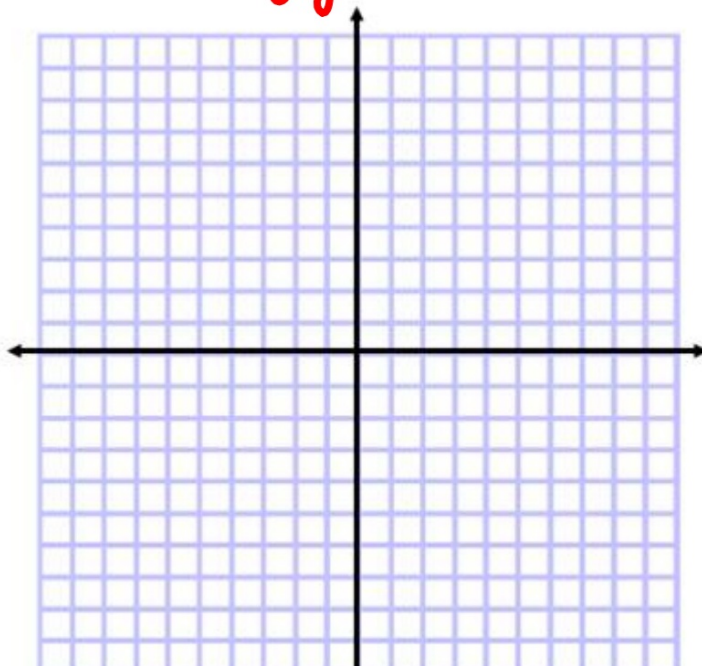
$$\sin T = \frac{1}{3}y$$

$$\cos^2 + \sin^2 = 1$$

$$\left(\frac{1}{2}x\right)^2 + \left(\frac{1}{3}y\right)^2 = 1$$

$$\frac{x^2}{4} + \frac{y^2}{9} = 1$$

what would be a window?



$$\cos^2 t + \sin^2 t = 1$$

Find parametric equations for each rectangular equation.

10. $y = 2x^2 - 5x$

$$x = 6 \cos t$$

$$y = 6 \sin t$$

11. $x^2 + y^2 = 36$

$$\cos t$$

$$\frac{x^2}{36} + \frac{y^2}{36} = 1$$

$$\frac{x}{6} = \cos t$$

$$x = 6 \cos t$$

$$x = t$$

$$y = 2t^2 - 5t$$

$$\sqrt{\frac{x^2}{36}} = \sqrt{\cos^2 t}$$

$$\sqrt{\frac{y^2}{36}} = \sqrt{\sin^2 t}$$

$$y = 6 \sin t$$

$$\frac{y}{6} = \sin t$$

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(radians t-step =0.05)