

Precalc 11.3

Use the exponential
function $y=e^x$

Quiz 11.1-11.2
tomorrow

exponential change

growth  .

decay 

irrational

e
compound interest

≈ 2.7

Exponential
Growth or
Decay
(in terms of e)

$$N = N_0 e^{kt}$$

where N is the final amount, N_0 is the initial amount, k is a constant and t is time.

$$A = A_0 \left(1 + \frac{r}{n}\right)^{nt}$$

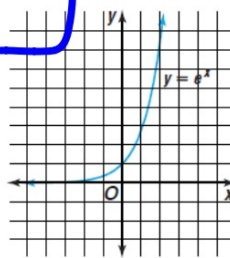
How to calculate e

The following computation for e is correct to three decimal places.

$$e = 1 + \frac{1}{1} + \frac{1}{1 \cdot 2} + \frac{1}{1 \cdot 2 \cdot 3} + \frac{1}{1 \cdot 2 \cdot 3 \cdot 4} + \frac{1}{1 \cdot 2 \cdot 3 \cdot 4 \cdot 5} + \frac{1}{1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 \cdot 6} + \frac{1}{1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 \cdot 6 \cdot 7}$$

$$\begin{aligned} &= 1 + 1 + \frac{1}{2} + \frac{1}{6} + \frac{1}{24} + \frac{1}{120} + \frac{1}{720} + \frac{1}{5040} \\ &= 1 + 1 + 0.5 + 0.16667 + 0.04167 + 0.00833 + 0.00139 + 0.000198 \\ &= 2.718 \end{aligned}$$

The function $y = e^x$ is one of the most important exponential functions. The graph of $y = e^x$ is shown at the right.



Continuously
Compounded
Interest

The equation $A = Pe^{rt}$, where P is the initial amount, A is the final amount, r is the annual interest rate, and t is time in years, is used for calculating interest that is compounded continuously.

Example



2

FINANCE

Compare the balance after 25 years of a \$10,000 investment earning 6.75% interest compounded continuously to the same investment compounded semiannually.

Semi

$$= 10,000 \left(1 + \frac{0.0675}{2}\right)^{2 \cdot 25}$$
$$= \$52,574.62$$

Cont'n

$$= 10000 e^{(0.0675 \cdot 25)}$$
$$= \$54,059.49$$

9. **Physics** Newton's Law of Cooling expresses the relationship between the temperature of a cooling object y and the time t elapsed since cooling began. This relationship is given by $y = ae^{-kt} + c$, where c is the temperature of the medium surrounding the cooling object, a is the difference between the initial temperature of the object and the surrounding temperature, and k is a constant related to the cooling object.

- The initial temperature of a liquid is 160°F . When it is removed from the heat, the temperature in the room is 76°F . For this object, $k = 0.23$. Find the temperature of the liquid after 15 minutes.
- Alex likes his coffee at a temperature of 135° . If he pours a cup of 170°F coffee in a 72°F room and waits 5 minutes before drinking, will his coffee be too hot or too cold? Explain. For Alex's cup, $k = 0.34$.

9-130M
15,16

$$= 84e^{-0.23 \cdot 15} + 160$$