Precalc 12.7

Approximate e^x trig values, and logs by using series
Use Euler's formula to write the exponential form of a complex number

Pascal's Triangle

Fibonacci sequence

exponential series calc with e

trigonometric series (radians) calc sin/cos/tan

Euler's formula

activity: sunflowers & pinecones

$(3x + -a^2)^5$ $-1 \cdot a^2$ $-a^2 \cdot -a^2$

$\cdot -a^2$

$1(3x)^5 \quad 5(3x)^4(-a^2)^1 \quad 10(3x)^3(-a^2)^2 \quad 10(3x)^2(a^2)^3 \quad 5(3x)^1(a^2)^4 \quad 1(a^2)^5$

$243x^5 - 405x^4a^2 + 270x^3a^4$

Fibonacci sequence

1, 1, 2, 3, 5, 8, 13 . . . 21 24

Fibonacci in Nature

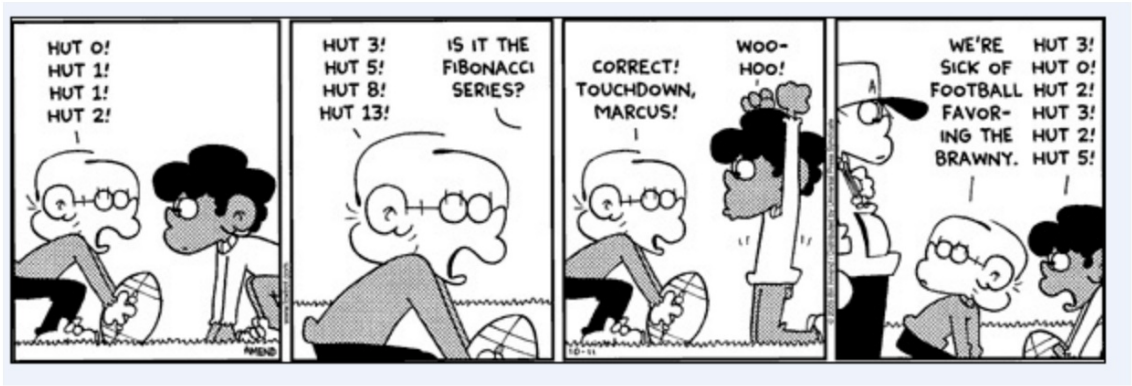
The number of petals on a flower is often a Fibonacci number.

n = 13

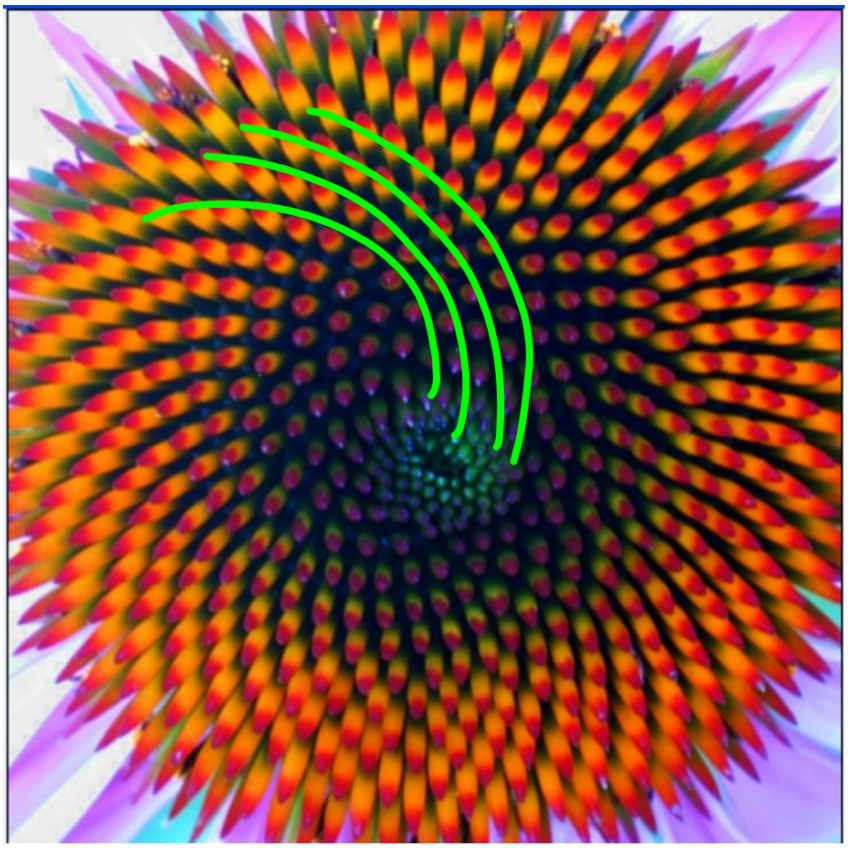


1 3 5 13

Why are four-leaf clovers so rare? Because 4 isn't a Fibonacci number!


HUT 0!
HUT 1!
HUT 1!
HUT 2!
HUT 3!
HUT 5!
HUT 8!
HUT 13!
IS IT THE FIBONACCI SERIES?
CORRECT! TOUCHDOWN, MARCUS!
WOO-HOO!
WE'RE SICK OF FOOTBALL FAVOR-ING THE BRAWNY.
HUT 3! HUT 0! HUT 2! HUT 3! HUT 2! HUT 5!

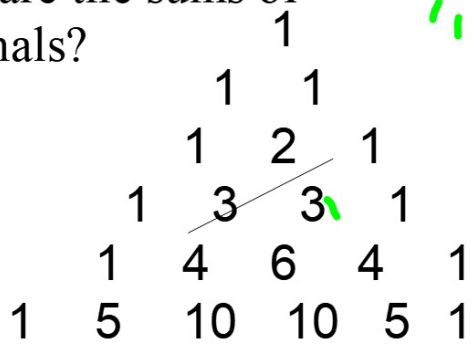
perrin nur

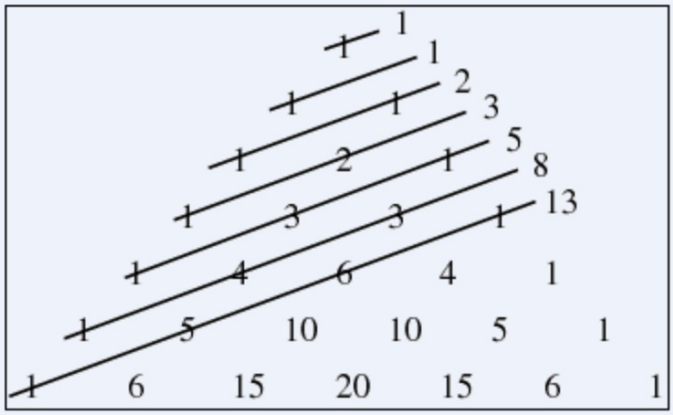


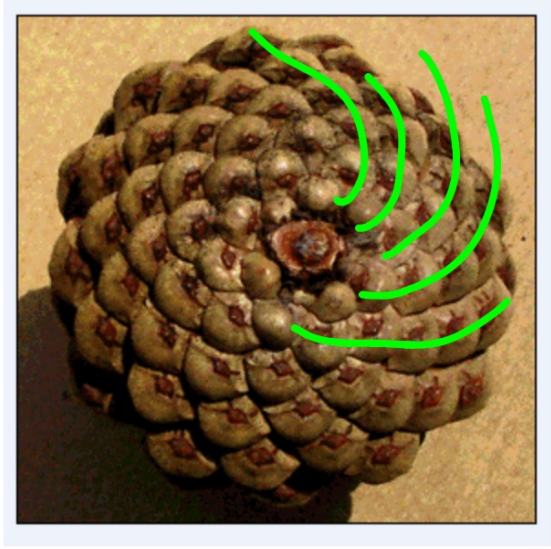
Pascals triangle

What are the sums of diagonals?

1, 1, 2, 3, 5

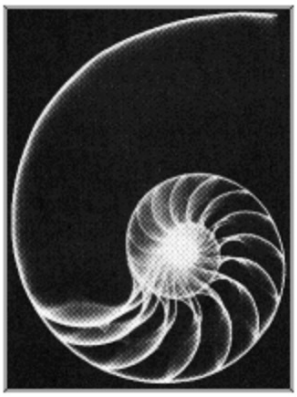



1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1
1
2
3
5
8
13









definition 0! = 1

Exponential Series	$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \frac{x^5}{5!} + \cdots$

e^x 7.61

Example 2 Use the first five terms of the exponential series and a calculator to approximate the value of $e^{2.03}$ to the nearest hundredth.

$e^{2.03} = 1 + 2.03 + \frac{(2.03)^2}{2!} + \frac{(2.03)^3}{3!} + \frac{(2.03)^4}{4!}$

$= 1 + 2.03 + 2.0605 + 1.3942 +$ 0.7076

$= 7.1923$

≈ 7.19

This is why your calculator has to "think" when you use e in calculations.

19-28 all

radian

Trigonometric Series

$$\cos x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!} = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \frac{x^8}{8!} - \cdots$$

$$\sin x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!} = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \frac{x^9}{9!} - \cdots$$

This is why your calculator has to "think about it" when you ask for sin, cos, etc.

$$\cos\frac{\pi}{3} = 1 - \frac{\left(\frac{\pi}{3}\right)^2}{2!} + \frac{\left(\frac{\pi}{3}\right)^4}{4!} - \frac{\left(\frac{\pi}{3}\right)^6}{6!} + \frac{\left(\frac{\pi}{3}\right)^8}{8!}$$

3 Use the first five terms of the trigonometric series to approximate the value of $\cos\frac{\pi}{3}$ to four decimal places.

$= 1 - 0.54831 + 0.05108 - 0.00183 + 0.00004$

.05011 .000029 .00036

$= 0.95098$

≈ 0.9510

=.05018
approx 0.5

Note: typo in original calculation

cis

Euler's Formula	$e^{i\alpha} = \cos\alpha + i\sin\alpha$

angle in radians
change to polar form

$$a + bi = r(\cos\theta + i\sin\theta)$$
$$= re^{i\theta}$$

ole **4** Write $1 + \sqrt{3}i$ in exponential form.

Not real!

$e^{i\Theta}=1\text{cis}\Theta$

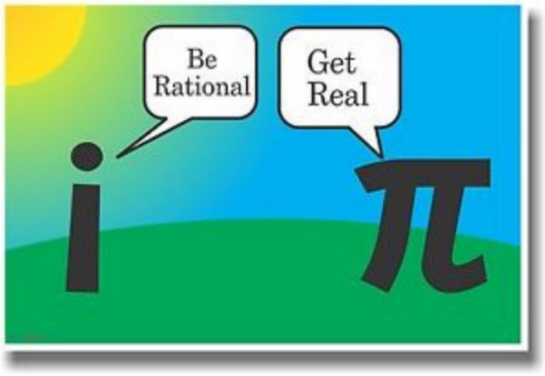
Can there be ln of a negative number?

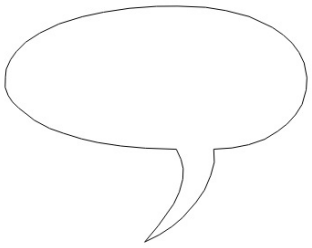
$\ln_e(\quad)$

just checking...

$$e^{i\alpha} = \cos\alpha + i\sin\alpha$$
$$e^{i\pi} = \cos\pi + i\sin\pi \quad \textit{Let } \alpha = \pi.$$
$$e^{i\pi} = -1 + i(0)$$
$$e^{i\pi} = -1 \quad \textit{So } e^{i\pi} + 1 = 0.$$

Wow, 3 of the most important numbers in one expression! (and also e & i)





-e

$e^{i\pi} = -1$

$$e^{i\pi} = -1$$

$$\ln e^{i\pi} = \ln(-1)$$
$$i\pi = \ln(-1)$$

5 **Evaluate ln (−270).**

+i

π