Precalc 12.2

Find the nth term and geometric means of a sequence
Find the sum of n terms of a geometric series

geometric sequence must not rule

difference (d)

2, 4, 6, 8 ...

common ratio (r)

2, 4,...?

2, 4, 8, 16 ...

geometric means

geome

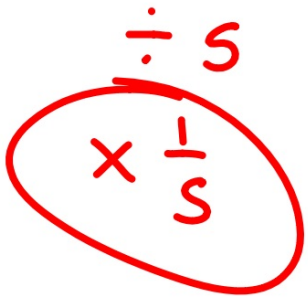
Geometric Sequence	A geometric sequence is a sequence in which each term after the first, a_1, is the product of the preceding term and the common ratio, r. The terms of the sequence can be represented as follows, where a_1 is nonzero and r is not equal to 1 or 0. $a_1, a_1r, a_1r^2, \ldots.$

3, 6, 12, 24, 48..

$r = 2$

The following sequence is an example of a **geometric sequence.**

10, 2, 0.4, 0.08, 0.016, … *Can you find the next term?*



Think in terms of mult (not division)

1 **Determine the common ratio and find the next three terms in each sequence.**

a. $1, -\frac{1}{2}, \frac{1}{4}, \ldots$

$-\frac{1}{2} \cdot ? = \frac{1}{4}$

$\frac{\frac{1}{4}}{-\frac{1}{2}} = -\frac{2}{4} = -\frac{1}{2}$

$r = -\frac{1}{2}$

$-\frac{1}{8}, \frac{1}{16}, -\frac{1}{32}$

1(?) = -1/2 -1/2(?) = 1/4

b. $(r-1), (-3r+3), (9r-9), \ldots$ a_1 a_2 a_3

$-27r+27$

$+81r-81$

$-243r+243$

$-3(-3r+3) \stackrel{?}{=} 9r-9$

$\frac{(-3r+3)}{-3r+3} \stackrel{?}{=} \frac{9r-9}{-3r+3} = \frac{3(3r-3)}{-1(3r-3)}$

$= -3$

first term	a_1
second term	$a_2 = a_1 r$
third term	$a_3 = a_1 r \cdot r = a_1 r^2$
fourth term	$a_4 = a_1 r^2 \cdot r = a_1 r^3$
fifth term	$a_5 = a_1 r^3 \cdot r = a_1 r^4$
⋮	⋮
nth term	$a_n = a_1 r^{(n-1)}$

The *n*th Term of a Geometric Sequence

The *n*th term of a geometric sequence with first term a_1 and common ratio r is given by $a_n = a_1 r^{n-1}$.

Example 2 **Find an approximation for the 23rd term in the sequence 256, −179.2, 125.44, ...**

$-179.2r = 125.44$

$r = -0.7$

$a_{23} = a_1 r^{22}$

$= 256(-0.7)^{22}$

$= 256(0.000391)$

≈ 0.1001

Geometric sequences can represent growth or decay.

- For a common ratio greater than 1, a sequence may model growth. Applications include compound interest, appreciation of property, and population growth.
- For a positive common ratio less than 1, a sequence may model decay. Applications include some radioactive behavior and depreciation.

a.( )

4 **Write a sequence that has two geometric means between 48 and −750.**

This sequence will have the form 48, ___?___, ___?___, −750.

48 -120 300 -750

a_1 a_2 a_3

$a_4 = a_1 r^3$

$-750 = 48 r^3$

$(-15.625) = (r^3)^{1(\frac{1}{3})}$

$-2.5 = r$

A **geometric series** is the indicated sum of the terms of a geometric sequence. The lists below show some examples of geometric sequences and their corresponding series.

Geometric Sequence	Geometric Series
$3, 9, 27, 81, 243$	$3 + 9 + 27 + 81 + 243$
$16, 4, 1, \frac{1}{4}, \frac{1}{16}$	$16 + 4 + 1 + \frac{1}{4} + \frac{1}{16}$
$a_1, a_2, a_3, a_4, \ldots, a_n$	$a_1 + a_2 + a_3 + a_4 + \cdots + a_n$

Sum of a Finite Geometric Series

The sum of the first n terms of a finite geometric series is given by $S_n = \frac{a_1 - a_1 r^n}{1 - r}$. $= \frac{a_1(1 - r^n)}{1 - r}$

Might be more helpful (later) to use the factored form instead:

Example 5 Find the sum of the first ten terms of the geometric series $16 - 48 + 144 - 432 + \cdots$.

$$S_n = \frac{a_1(1 - r^n)}{1 - r}$$

$$= \frac{16(1 - (-3)^{10})}{1 + 3} = \frac{16(1 - 59049)}{4}$$

$$= -236192$$

Lesson 12-1 *(Pages 759–765)*

Find the next four terms in each arithmetic sequence.

1. $7, 3, -1, \ldots$
2. $0.5, -1, -2.5, \ldots$
4. $3, 2.8, 2.6, \ldots$
5. $4x, -x, -6x, \ldots$

For Exercises 7–13, assume that each sequence or series is arithmetic.

7. Find the 16th term in the sequence for which $a_1 = 2$ and $d = 5$.

10. Find d for the sequence in which $a_1 = 7$ and $a_{13} = 30$.

12. Find the sum of the first 12 terms in the series $2 + 2.8 + 3.6 + \cdots$.

$+0.8$

$\frac{12}{2}\left(2 + \overset{10.8}{?}\right) = 76.8$

17-41 odd

$a_{12} = a_1 + 11d$

$= 2 + 11(.8)$

$= 10.8$

Lesson 12-2 *(Pages 766–773)*

Determine the common ratio and find the next three terms of each geometric sequ

1. $14, 7, 3.5, \ldots$
2. $-2, 4, -8, \ldots$
3. $\frac{2}{3}, \frac{1}{2}, \frac{3}{8}, \ldots$
4. $10, -5, 2.5, \ldots$
5. $8, 8\sqrt{2}, 16, \ldots$
6. $a^{10}, a^8, a^6, \ldots$

For Exercises 7–11, assume that each sequence or series is geometric.

7. Find the sixth term of a sequence whose first term is 9 and common ratio is 2.

8. If $r = 4$ and $a_8 = 100$, what is the first term of the sequence?

10. Write a sequence that has two geometric means between 4 and 256.

11. What is the sum of the first six terms of the series $3 + 9 + 27 + \cdots$?