

Geometry 2.3

Analyze statements in if-then form

Write the converse, inverse, and contrapositive of conditional statements

	$\neg p \rightarrow q$	2 ways	
conditional statement		biconditional	
hypothesis		"if and only if"	
conclusion		iff	$p \rightarrow q$ OG
related conditional			$q \rightarrow p$ C
converse	$q \rightarrow p$	T	
inverse	$\neg p \rightarrow \neg q$	F	
contrapositive	$\neg q \rightarrow \neg p$	T	
logically equivalent			

KeyConcept Conditional Statement

Words	Symbols
An if-then statement is of the form <i>if p, then q</i> .	$p \rightarrow q$ read <i>if p then q</i> , or <i>p implies q</i>
The hypothesis of a conditional statement is the phrase immediately following the word <i>if</i> .	p
The conclusion of a conditional statement is the phrase immediately following the word <i>then</i> .	q

2 Related Conditionals There are other statements that are based on a given conditional statement. These are known as **related conditionals**.

KeyConcept Related Conditionals 		
Words	Symbols	Examples
A conditional statement is a statement that can be written in the form <i>if p, then q</i> .	$p \rightarrow q$	If $m\angle A$ is 35, then $\angle A$ is an acute angle.
The converse is formed by exchanging the hypothesis and conclusion of the conditional.	$q \rightarrow p$	If $\angle A$ is an acute angle, then $m\angle A$ is 35.
The inverse is formed by negating both the hypothesis and conclusion of the conditional.	$\sim p \rightarrow \sim q$	If $m\angle A$ is <i>not</i> 35, then $\angle A$ is <i>not</i> an acute angle.
The contrapositive is formed by negating both the hypothesis and the conclusion of the converse of the conditional.	$\sim q \rightarrow \sim p$	If $\angle A$ is <i>not</i> an acute angle, then $m\angle A$ is <i>not</i> 35.

If you live in Sioux Falls, then you live in SD.

A conditional and its contrapositive are either both true or both false. Similarly, the converse and inverse of a conditional are either both true or both false. Statements with the same truth values are said to be **logically equivalent**.

KeyConcept Logically Equivalent Statements

- A conditional and its contrapositive are logically equivalent.
- The converse and inverse of a conditional are logically equivalent.

Example 3 Determine the truth value of each conditional statement. If *true*, explain your reasoning. If *false*, give a counterexample.

10. If $x^2 = 16$, then $x = 4$.
11. If you live in Charlotte, then you live in North Carolina.
12. If tomorrow is Friday, then today is Thursday.
13. If an animal is spotted, then it is a Dalmatian.
14. If the measure of a right angle is 95, then bees are lizards.
15. If pigs can fly, then $2 + 5 = 7$.

Remember: benefit of the doubt...

$S \rightarrow D \quad F$
 $(C) D \rightarrow S \quad T$
 $(I) \sim S \rightarrow \sim D \quad T$
 $F \text{ hyp.}$
 $(CP) \sim D \rightarrow \sim S \quad F$

Example 4



ARGUMENTS Write the converse, inverse, and contrapositive of each true conditional statement. Determine whether each related conditional is *true* or *false*. If a statement is false, find a counterexample.

16. If a number is divisible by 2, then it is divisible by 4.
17. All whole numbers are integers



Start by writing in if/then form.

$$\frac{180}{3}$$

If a triangle is equilateral, then each angle is 60 degrees. T

If a triangle has three 60 degree angles, then it is equilateral. T

"If and only if...iff"

A Δ is equilateral iff each $\angle = 60^\circ$

T
 T
 T
 T

Key Concept Biconditional Statement

Words

A biconditional statement is the conjunction of a conditional and its converse.

Symbols

$(p \rightarrow q) \wedge (q \rightarrow p) \rightarrow (p \leftrightarrow q)$, read p *if and only if* q

If and only if can be abbreviated iff.

Examples

Write each biconditional as a conditional and its converse. Then determine whether the biconditional is *true* or *false*. If false, give a counterexample.

- a. An angle is a right angle if and only if its measure is 90.

Conditional: If an angle measures 90, then the angle is right.

Converse: If an angle is right, then the angle measures 90.

Both the conditional and the converse are true, so the biconditional is true.

- b. $x > -2$ iff x is positive.

Conditional: If x is positive, then $x > -2$.

Converse: If $x > -2$, then x is positive.

Let $x = -1$. Then $-1 > -2$, but -1 is not positive. So, the biconditional is false.

or if $r + \angle$ then 90° T

c if 90° then $r + \angle$ T

if $x > -2$ then x pos. F ex $x = -1$
if x pos then $x > -2$ T

F

Write both statement & converse. Are they both true?

Exercises

Write each biconditional as a conditional and its converse. Then determine whether the biconditional is *true* or *false*. If false, give a counterexample.

1. Two angles are complement if and only if their measures have a sum of 90.
2. There is no school if and only if it is Saturday.

if comp then sum 90 T

if sum 90 then comp T

if Sat then No Sch T

if no school then Sat F

Project work time/HW
Due date has moved to Monday 3:30

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