

Algebra 2 3.4

Solve systems of linear equations in 3 variables

Use systems to solve problems

ordered triple (x, y, z)

no solution

infinitely many solutions

substitution

☺ elimination

put in something else =
make zero pairs

whiteboards? if time

Can you solve this?

$$\text{🌮} + \text{🌮} + \text{🌮} = 60 \quad \text{1 taco} = 20$$

$$\text{🌮}^{20} + \text{🌯} + \text{🌯} = 30$$

$$\text{🌯}^5 - \text{🌶️} = 3$$

$$\text{🌶️} + \text{🌮} + \text{🌯} = ?$$

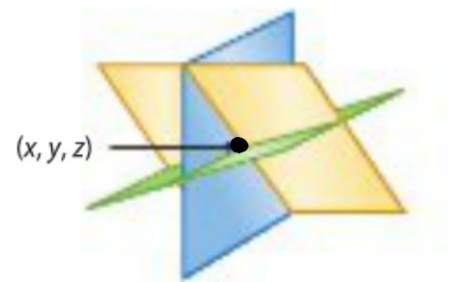
$$\begin{array}{rclcl}
 \text{Apple} & + & \text{Apple} & + & \text{Apple} & = & 60 \\
 \text{Apple} & + & \text{Orange} & + & \text{Orange} & = & 40 \\
 \text{Orange} & - & \text{Pineapple} & = & 4 \\
 \text{Pineapple} & + & \text{Apple} & \times & \text{Orange} & = & ?
 \end{array}$$

A = 20

3 x 100

One Solution

The three individual planes intersect at a specific point.

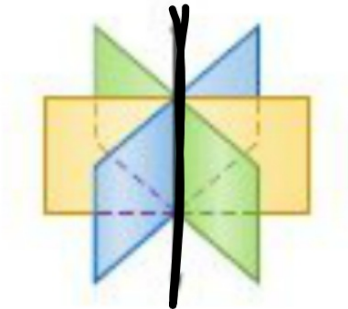


0=0
5=12

Infinitely Many Solutions

The planes intersect in a line.

Every coordinate on the line represents a solution of the system.



The planes intersect in the same plane.

Every equation is equivalent.

Every coordinate in the plane represents a solution of the system.



Don't worry about "which" infinitely many it is...

No Solution There are no points in common with all three planes.



We won't worry about *which* no solution it is.

$$(x, y, z)$$

$$(-1, -5, 7)$$

$$a) \begin{array}{r} 3 \cdot -1 - 2y + 4 \cdot 7 = 5 \\ -3 - 2y + 28 \\ -2y + 25 \\ -2y + 25 \\ \hline -2y = -25 \end{array}$$

Be strategic!

1. elimination

2. elim again

(or could use subs)

3. ans. is ordered triple

2x the same thing

$$\Downarrow \downarrow + -5 - 35 = -36$$

$$-36 = -36$$

Example 1 A System with One Solution

Solve the system of equations.

$$\begin{array}{l} a) \quad 3x - 2y + 4z = 35 \\ b) \quad -4x + y - 5z = -36 \\ c) \quad 5x - 3y + 3z = 31 \end{array}$$

The coefficient of 1 in the second equation makes y a good choice for elimination.

$$\begin{array}{r} 3x - 2y + 4z = 35 \\ -8x + 2y - 10z = -72 \\ \hline \end{array}$$

a+b elimy
b+c elimy

$$\begin{array}{r} -5x - 6z = -37 \\ -7x - 12z = -77 \end{array}$$

$$\begin{array}{r} 10x + 12z = 74 \\ -7x - 12z = -77 \\ \hline \end{array}$$

$$\frac{3x}{3} = -\frac{3}{3} \quad x = -1$$

$$\begin{array}{r} -12x + 3y - 15z = -108 \\ 5x - 3y + 3z = 31 \\ \hline \end{array}$$

$$-7x - 12z = -77$$

$$-7 \cdot -1 - 12z = -77$$

$$7 - 12z = -77$$

$$\begin{array}{r} -7 \\ -12z = -84 \end{array} \quad z = 7$$

$$(x, y, z) = (3, -2, -4)$$

Guided Practice

Strategy: elim is usually a good place to start

$$a) \quad 2x + 4y + 5z = 18$$

$$b) \quad -3x + 5y + 2z = -27$$

$$c) \quad -5x + 3y - z = -17 \rightarrow \cdot 5$$

$$b + c \downarrow$$

$$a + c \downarrow$$

$$elim \ z$$

$$-13x + 11y = -61$$

$$27x - 11y = 103$$

$$-13x + 11y = -61$$

$$27x - 11y = 103$$

$$\frac{14x}{14} = \frac{42}{14} \quad x = 3$$

$$-3x + 5y + 2z = -27$$

$$-10x + 6y - 2z = -34$$

$$-13x + 11y = -61$$

$$25x - 15y + 5z = 85$$

$$2x + 4y - 5z = 18$$

$$27x - 11y = 103$$

$$27 \cdot 3 - 11y = 103$$

$$81 - 11y = 103$$

$$-11y = 22$$

$$y = -2$$

#variables = # equations

- Seats closest to an amphitheater stage cost \$30. The seats in the next section cost \$25, and lawn seats are \$20. There are twice as many seats in section B as in section A. When all 19,200 seats are sold, the amphitheater makes \$456,000.

A system of equations in three variables can be used to determine the number of seats in each section.



Whiteboards

1B. $4x - 3y + 6z = 18$
 $-x + 5y + 4z = 48$
 $6x - 2y + 5z = 0$

Example 2 No Solution and Infinite Solutions

Solve each system of equations.

a. $5x + 4y - 5z = -10$
 $-4x - 10y - 8z = -16$
 $6x + 15y + 12z = 24$

b. $-6a + 9b - 12c = 21$
 $-2a + 3b - 4c = 7$
 $10a - 15b + 20c = -30$

Guided Practice

2A. $-4x - 2y - z = 15$
 $12x + 6y + 3z = 45$
 $2x + 5y + 7z = -29$

2B. $3x + 5y - 2z = 13$
 $-5x - 2y - 4z = 20$
 $-14x - 17y + 2z = -19$

