Algebra 2 3.8

Find the inverse of a 2x2 matrix

Write and solve matrix equations

$$2x + 3 = 11$$
$$-3 \quad -3$$
$$\frac{2x}{2} = \frac{8}{2}$$
$$x = 4$$

square matrix 2×2
3×3

identity matrix

inverse matrix

matrix equation

coefficient matrix

variable matrix

constant matrix

whiteboards

Matrix equation is different than Cramer's rule...

(Most students have a favorite after they try both methods.)
Which one do you lke?

Wife Swap (TV?)

2 × 2 Identity Matrix

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

3 × 3 Identity Matrix

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$A \quad B$$

$$\begin{bmatrix} 4 & 5 \\ 3 & 4 \end{bmatrix} \cdot \begin{bmatrix} 4 & -5 \\ -3 & 4 \end{bmatrix} =$$

13-35 odd
3.8 p. 202

KeyConcept Identity Matrix for Multiplication

Words The identity matrix for multiplication I is a square matrix with 1 for every element of the main diagonal, from upper left to lower right, and 0 in all other positions. For any square matrix A of the same dimension as I, $A \cdot I = I \cdot A = A$.

Symbols If $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, then $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ such that

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}.$$

$\frac{x}{3} \cdot 3 = x$

Two $n \times n$ matrices are **inverses** of each other if their product is the identity matrix. If matrix A has an inverse symbolized by A^{-1}, then $A \cdot A^{-1} = A^{-1} \cdot A = I$.

$$\begin{bmatrix} & \\ & \end{bmatrix}_{A} \cdot \begin{bmatrix} & \\ & \end{bmatrix}_{A^{-1}} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Is their product the identity matrix?

Example 1 Verify Inverse Matrices

Determine whether the matrices in each pair are inverses.

a. $A = \begin{bmatrix} -4 & 2 \\ -2 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} \frac{1}{4} & -\frac{1}{2} \\ \frac{1}{2} & -1 \end{bmatrix}$

$$\begin{bmatrix} -4 & 2 \\ -2 & 1 \end{bmatrix} \cdot \begin{bmatrix} \frac{1}{4} & -\frac{1}{2} \\ \frac{1}{2} & -1 \end{bmatrix} = \begin{bmatrix} 0 & \\ & \end{bmatrix}$$

Determine whether the matrices in each pair are inverses.

1. $A = \begin{bmatrix} 2 & 1 \\ -1 & 0 \end{bmatrix}, B = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$

2. $C = \begin{bmatrix} 2 & 1 \\ 5 & 3 \end{bmatrix}, D = \begin{bmatrix} 2 & 1 \\ 5 & -3 \end{bmatrix}$

$\begin{bmatrix} 2 & 1 \\ -1 & 0 \end{bmatrix} \cdot \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} 4$

KeyConcept Inverse of a 2 × 2 Matrix

The inverse of matrix $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is $A^{-1} = \frac{1}{ad - bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$, where $ad - bc \neq 0$.

$$\begin{bmatrix} 3 & 2 \\ 0 & 5 \end{bmatrix}$$

0

15 − 0

15

What is ad-bc?
1/det

$$\frac{1}{15}\begin{bmatrix} 5 & -2 \\ 0 & 3 \end{bmatrix}$$

Wife Swap: TV show

Example 2 Find the Inverse of a Matrix

Find the inverse of each matrix, if it exists.

a. $P = \begin{bmatrix} 7 & -5 \\ 2 & -1 \end{bmatrix}$

-10

$-7+10$

-7

$\frac{1}{\det} \begin{bmatrix} -1 & 5 \\ -2 & 7 \end{bmatrix}$

$\frac{1}{3}\begin{bmatrix} -1 & 5 \\ -2 & 7 \end{bmatrix}$

Why would an inverse not exist?

b. $Q = \begin{bmatrix} -8 & -6 \\ 12 & 9 \end{bmatrix}$ $-72 \quad -72 - -72 = 0$

-72

$\frac{1}{0}$ NP

whiteboards

GuidedPractice

2A. $\begin{bmatrix} 3 & 7 \\ 1 & -4 \end{bmatrix}$ $-12-7$

-12

$\frac{1}{-19}$

$\frac{-1}{19}\begin{bmatrix} -4 & -7 \\ -1 & 3 \end{bmatrix}$

2B. $\begin{bmatrix} 2 & 1 \\ -4 & 3 \end{bmatrix}$

Find the inverse of each matrix, if it exists.

5. $\begin{bmatrix} 6 & -3 \\ -1 & 0 \end{bmatrix}$

6. $\begin{bmatrix} 2 & -4 \\ -3 & 0 \end{bmatrix}$

Different than Cramer's rule...

$\frac{-60}{-12}$ $\frac{-24}{-12}$ $x=5$ $y=2$

2 Matrix Equations

Matrices can be used to represent and solve systems of equations. You can write a **matrix equation** to solve the system of equations below.

$$\begin{aligned} x + 2y &= 9 \\ 3x - 6y &= 3 \end{aligned} \rightarrow \begin{bmatrix} x + 2y \\ 3x - 6y \end{bmatrix} = \begin{bmatrix} 9 \\ 3 \end{bmatrix}$$

coefficient matrix
variable matrix
constant matrix

$\begin{vmatrix} 1 & 2 \\ 3 & -6 \end{vmatrix}$ $+6$ $-6-6$

$x = \begin{vmatrix} 9 & 2 \\ 3 & -6 \end{vmatrix}$ 6 $-54-6=-60$ -54

To solve: $A^{-1}xB$

$y = \begin{vmatrix} 1 & 9 \\ 3 & 3 \end{vmatrix}$ 27 $3-27$ -24 3

$$x + 2y = 9$$
$$3x - 6y = 3$$

$$\underbrace{\begin{bmatrix} 1 & 2 \\ 3 & -6 \end{bmatrix}}_{A} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \underbrace{\begin{bmatrix} 9 \\ 3 \end{bmatrix}}_{B}$$

$A^{-1} \cdot B$

$2 \times 2 \quad 2 \times 1$

Use a matrix equation to solve each system of equations.

9. $-2x + y = 9$
$x + y = 3$

10. $4x - 2y = 22$
$6x + 9y = -3$

$\begin{bmatrix} 4 \\ -3 \end{bmatrix}$

$x = \begin{bmatrix} -2 \\ 5 \end{bmatrix}$
$y = \begin{bmatrix} -2 \\ 5 \end{bmatrix}$

$A \quad B$

$\begin{bmatrix} -2 & 1 \\ 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 9 \\ 3 \end{bmatrix}$ $A^{-1} \cdot B$

$$-9x + 5y + 3z = 50$$
$$7x + 8y - 2z = -60$$
$$-5x + 7y + 5z = 46$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix}$$