Algebra 2 3.7

Evaluate determinants

Use Cramer's rule to solve systems of equations

Quiz 3.5-3.6 Wed.

matrix r × c

square matrix r = c

determinant only sq matrix

second order determinant

third order determinant

diagonal method, cofactor method, echelon method

Cramer's rule

coefficient matrix

$$\begin{bmatrix} - & - & - \\ - & - & - \end{bmatrix}$$

2 × 3

whiteboards

KeyConcept Second-Order Determinant

Words	The value of a second-order determinant is the difference of the products of the two diagonals.
Symbols	$\det \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$
Example	$\begin{vmatrix} 4 & 5 \\ -3 & 6 \end{vmatrix} = 4(6) - (5)(-3) = 39$

Notice how the notation has changed...
Does it look like something else?
context...

$A = \begin{bmatrix} 4 & 5 \\ -3 & 6 \end{bmatrix}$

$\begin{vmatrix} 4 & 5 \\ -3 & 6 \end{vmatrix}$ -15 24

$= 24 - -15$

$24 + 15$

39



Example 1 Second-Order Determinant

Evaluate each determinant.

a. $\begin{vmatrix} 5 & -4 \\ 8 & 9 \end{vmatrix}$ -32 45 $45 - -32 = 77$

b. $\begin{vmatrix} 0 & 6 \\ 4 & -11 \end{vmatrix}$ 24 0 $0 - 24$

GuidedPractice

1A. $\begin{vmatrix} -6 & -7 \\ 10 & -8 \end{vmatrix}$

-70

-48

$-48 + 70$

1B. $\begin{vmatrix} 7 & -5 \\ 9 & -1 \end{vmatrix}$

45

-26

$-28 - 45$

-73

There are multiple methods to evaluate a determinant for 3x3
(We are going to learn the diagonal method...for now.)

Example 2 Use Diagonals

Evaluate $\begin{vmatrix} 4 & -8 & 3 \\ -3 & 2 & 6 \\ -4 & 5 & 9 \end{vmatrix}$ using diagonals.

$-24 + 120 + 216 = 312$



$-165 + 312 = -477$

$72 \quad -192 \quad -45 = -165$

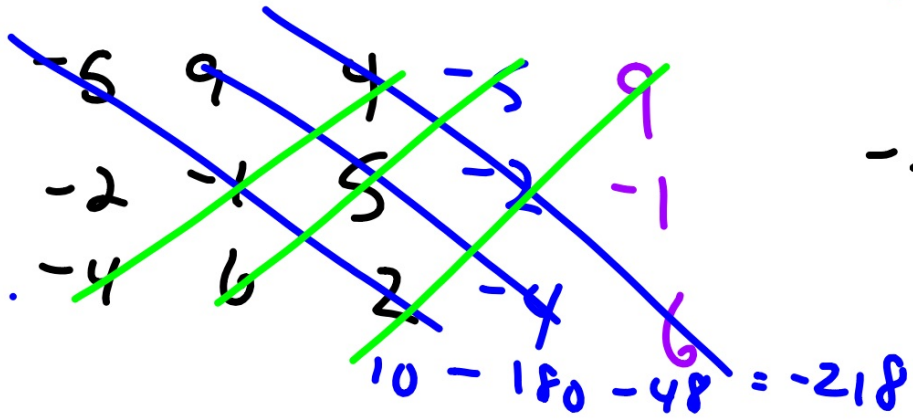
Guided Practice

Evaluate each determinant.

2A. $\begin{vmatrix} -5 & 9 & 4 \\ -2 & -1 & 5 \\ -4 & 6 & 2 \end{vmatrix}$

2B. $\begin{vmatrix} -8 & -4 & 4 \\ 0 & -5 & -8 \\ 3 & 4 & 1 \end{vmatrix}$

16 − 150 − 36 = −170



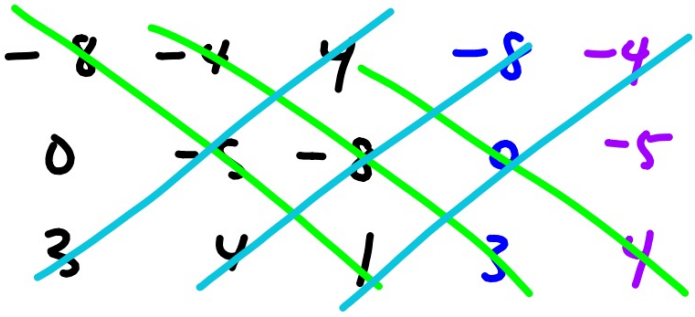
10 − 180 − 48 = −218

−218 − −170

−218 + 170

−48

$-60 + 256 + 0 = 196$



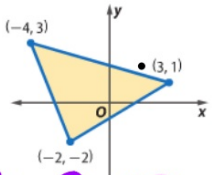
$136 - 196 = -60$

$40 + 96 + 0 = 136$

KeyConcept Area of a Triangle

Words The area of a triangle with vertices (a, b), (c, d), and (e, f) is $|A|$, where

$$A = \frac{1}{2}\begin{vmatrix} a & b & 1 \\ c & d & 1 \\ e & f & 1 \end{vmatrix}.$$



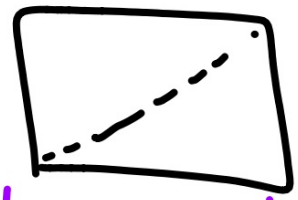
Example

$$A = \frac{1}{2}\begin{vmatrix} -4 & 3 & 1 \\ 3 & 1 & 1 \\ -2 & -2 & 1 \end{vmatrix}$$

$2 + 8 + 9 \quad 19$

$\frac{1}{2} \cdot |-35|$

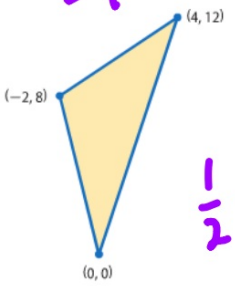
$A = \frac{1}{2}bh$



$\frac{1}{2} \cdot \begin{vmatrix} -4 & 3 & 1 \\ 3 & 1 & 1 \\ -2 & -2 & 1 \end{vmatrix}$

$-4 \quad -6 \quad -6 = -16$

$\frac{1}{2} \cdot 35 = 17.5$



$\frac{1}{2} \cdot \begin{vmatrix} 4 & 12 & 1 \\ -2 & 8 & 1 \\ 0 & 0 & 1 \end{vmatrix}$

KeyConcept Cramer's Rule

Let C be the coefficient matrix of the system $\begin{aligned} ax + by &= m \\ fx + gy &= n \end{aligned}$ $\rightarrow$ $\begin{bmatrix} a & b \\ f & g \end{bmatrix}$.

The solution of this system is $x = \dfrac{\begin{vmatrix} m & b \\ n & g \end{vmatrix}}{|C|}$ and $y = \dfrac{\begin{vmatrix} a & m \\ f & n \end{vmatrix}}{|C|}$, if $|C| \neq 0$.

coefficient matrix
variable matrix (either x or y)

Example 4 Solve a System of Two Equations

Solve the system by using Cramer's Rule.

$5x - 6y = 15$

$3x + 4y = -29$

$\begin{bmatrix} -3 \\ -5 \end{bmatrix}$

$\begin{vmatrix} 5 & -6 \\ 3 & 4 \end{vmatrix}$ -18

20

$20 + 18$

$x = -3$ $\quad (-3, -5)$

$x = \frac{-114}{38}$ $\quad y = \frac{-190}{38}$ $\quad y = -5$

$x = \begin{vmatrix} 15 & -6 \\ -29 & 4 \end{vmatrix}$ $\quad 174$

$60 - 174$

45

60

$y = \begin{vmatrix} 5 & 15 \\ 3 & -29 \end{vmatrix}$ $\quad -145 - 45$

-145

Test-Taking Tip

Cramer's Rule When the determinant of the coefficient matrix C is 0, the system does not have a unique solution.

Zero in the denominator...

$6x - 5y = 73$

$-7x + 3y = -71$

$\begin{vmatrix} 6 & -5 \\ -7 & 3 \end{vmatrix}$ 35

$18 - 35 =$

18

$x = 9$ $\qquad$ $y = -5$

$x = \frac{-136}{-17}$ $\qquad$ $y = \frac{85}{-17}$

$x = \begin{vmatrix} 73 & -5 \\ -71 & 3 \end{vmatrix}$ 355

$219 - 355$

219

$y = \begin{vmatrix} 6 & 73 \\ -7 & -71 \end{vmatrix}$ -511

$-426 + 511 = 85$

-426

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