

Algebra 2 3.3

Find the maximum and minimum values of a function over a region

Solve optimization problems using linear programming

maximum *biggest*

minimum *smallest*

constraints *requirement*

feasible region

bounded

unbounded

optimize *best solution*

linear programming

objective function

$$1. \quad \begin{aligned} \textcircled{x} &= 1^{\text{st}} \text{ no.} = 14 \\ \textcircled{y} &= 2^{\text{nd}} \text{ no.} = 7 \end{aligned}$$

$$2. \quad \begin{aligned} x &= 1^{\text{st}} \quad 6 \\ y &= 2^{\text{nd}} \quad -4 \end{aligned} \quad \begin{aligned} x + y &= 2 \\ x &= (y + 10) \end{aligned}$$

$$\begin{aligned} y + 10 + y &= 2 \\ 2y + 10 &= 2 \\ \underline{-10} \quad \underline{-10} \\ 2y &= -8 \\ y &= -4 \end{aligned}$$

$$x + y = 21$$

$$x = 2y$$

$$2y + y = 21$$

$$\frac{3y}{3} = \frac{21}{3}$$

$$x + y = -9$$

$$x = 2y$$

$$\begin{array}{l} 13 \text{ x } 1^{\text{st}} \\ 18 \text{ y } 2^{\text{nd}} \end{array}$$

$$\begin{array}{r} 1 \ 2 \ 3 \ 4 \\ 2 \ 1 \ 7 \\ 3 \ 7 \ 8 \\ 4 \\ 6 \ 3 \ 9 \end{array}$$

$$x = y - 5$$

$$9 \ 2 \ 6$$

$$13 \ 1 \ 8$$

$$y - 18 = 0$$

$$\begin{array}{r} -234 \\ -5 \end{array}$$

$$xy = 234$$

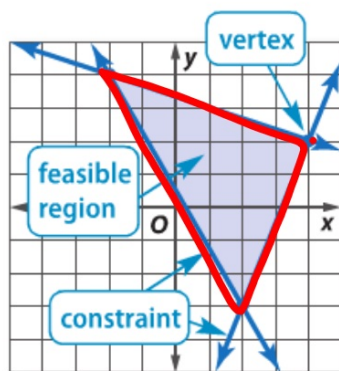
$$(y - 5)y = 234$$

$$y^2 - 5y - 234 = 0$$

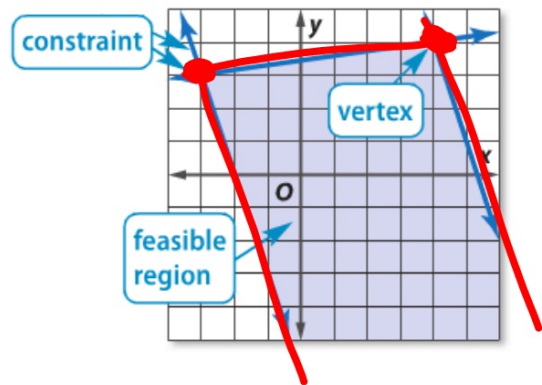
$$(y - 18)(y + 13) = 0$$

$$y = 18 \quad y = -13$$

KeyConcept Feasible Regions



The feasible region is enclosed, or **bounded**, by the constraints. The maximum or minimum value of the related function *always* occurs at a vertex of the feasible region.



The feasible region is open and can go on forever. It is **unbounded**. Unbounded regions have either a maximum or a minimum.

What does it mean when we say
"The object of the game is..."

goal

" objective function "

$f(x, y)$

Use each vertex in the objective function...max and/or min?

Example 1 Bounded Region

Graph the system of inequalities. Name the coordinates of the vertices of the feasible region. Find the maximum and minimum values of the function for this region.

$$3 \leq y \leq 6$$

$$0 \leq x + 12$$

$$0 \leq -x + 6$$

$$f(x, y) = 4x - 2y$$

$$y = 3x + 2$$

$$f(x, y) = 4x - 2y$$

min

$$(-3, 3) = -12 - 6 = -18$$

Which one is different?

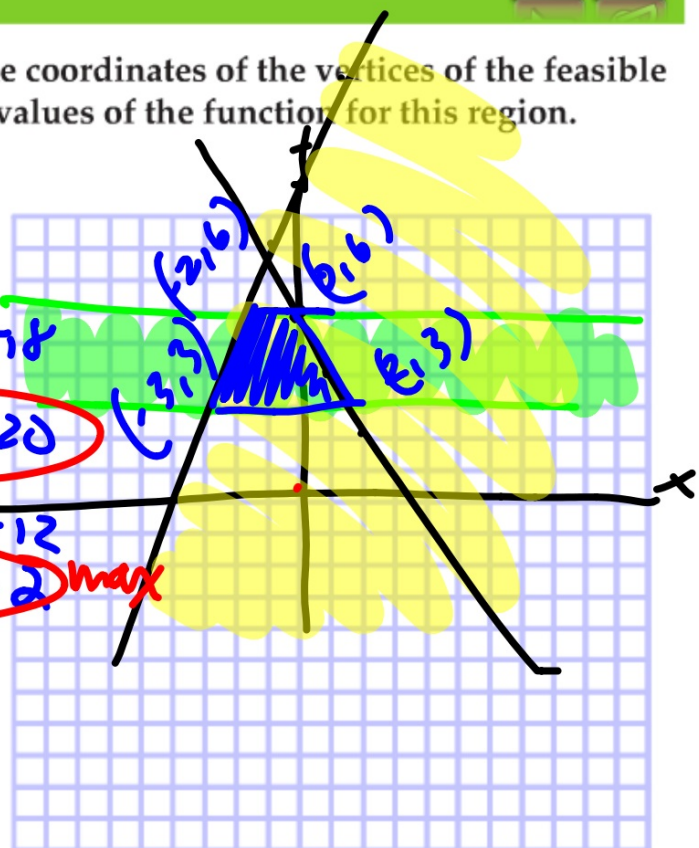
$$(-2, 6) = -8 - 12 = -20$$

The "objective function" is not part of the graph.

It is the object of the game.

$$(0, 6) = 0 - 12 = -12$$

$$(2, 3) = 8 - 6 = 2 \text{ max}$$



$$2y + 3x = -12$$

$$-3x = -3x$$

$$2y = -3x - 12$$

$$y = -\frac{3}{2}x - 6$$

Example 2 Unbounded Region

Graph the system of inequalities. Name the coordinates of the vertices of the feasible region. Find the maximum and minimum values of the function for this region.

$$x + y \geq -12$$

$$y \leq 3x + 12$$

$$y \geq 3x - 6$$

$$f(x, y) = 9x - 6y$$

$$y = 3x - 6$$

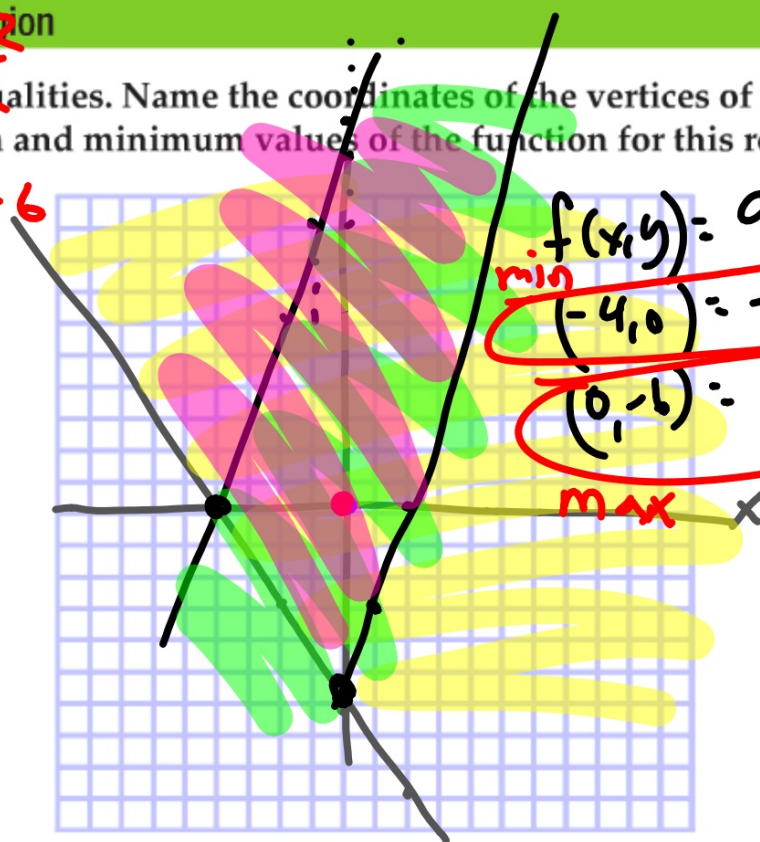
$$(-4, 0)$$

$$(0, -6)$$

$$f(x, y) = 9x - 6y$$

$$\text{min } (-4, 0) = -36 - 0 = -36$$

$$\text{max } (0, -6) = 0 + 36 = 36$$



9-21000
p. 156
2 Optimization To **optimize** means to seek the best price or amount to minimize costs or maximize profits. This is often obtained with the use of linear programming.

KeyConcept Optimization with Linear Programming



- Step 1** Define the variables.
- Step 2** Write a system of inequalities.
- Step 3** Graph the system of inequalities.
- Step 4** Find the coordinates of the vertices of the feasible region.
- Step 5** Write a linear function to be maximized or minimized.
- Step 6** Substitute the coordinates of the vertices into the function.
- Step 7** Select the greatest or least result. Answer the problem.

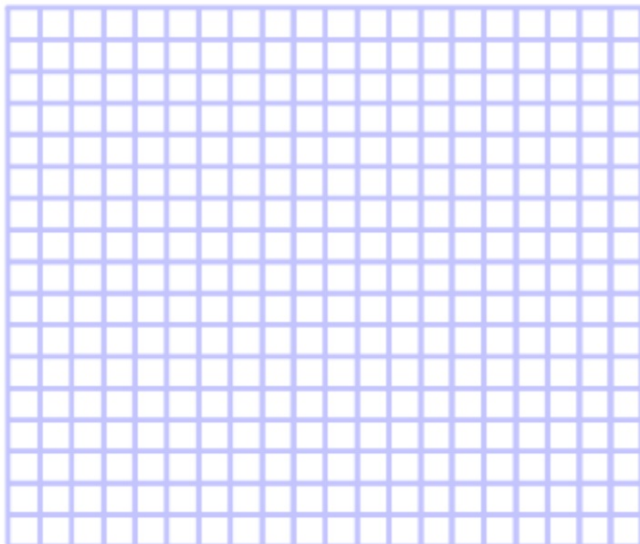
"objective function"

Guided Practice

Can she make a negative number of earrings?

3. **JEWELRY** Each week, Mackenzie can make 10 to 25 necklaces and 15 to 40 pairs of earrings. If she earns profits of \$3 on each pair of earrings and \$5 on each necklace, and she plans to sell at least 30 pieces of jewelry, how can she maximize profit?
-

What is the object of the game?
Write the objective function first.



Real-World Example 3 Optimization with Linear Programming



BUSINESS Refer to the application at the beginning of the lesson. Determine how many of each type of device should be made per shift.

What is the objective function?

Write that one first...

graph

answer the question

Why?

- An electronics company produces digital audio players and phones. A sign on the company bulletin board is shown.

If at least 2000 items must be produced per shift, how many of each type should be made to minimize costs?

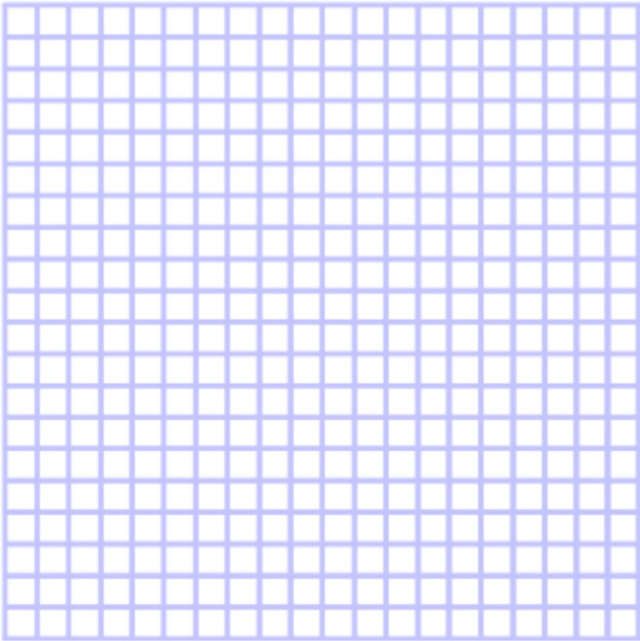
The company is experiencing limitations, or constraints, on production caused by customer demand, shipping, and the productivity of their factory. A system of inequalities can be used to represent these constraints.

Keeping Costs Down: We Can Do It!

Our Goal: Production per Shift			
Unit	Minimum	Maximum	Cost per Unit
audio	600	1500	\$55
phone	800	1700	\$95



phone



audio

What is the objective function?

