Trig 4.1

Determine roots of polynomial equations *
Apply the fundamental theorem of algebra (# of roots) *

degree $3x^4 + 5x^3 - 2x + 18x^2 = 0$

leading coefficient $f(x) =$

polynomial function $y =$

zeros (real) $= 0$

*Algebra 2 Ch. 5 & Ch. 7

polynomial equation

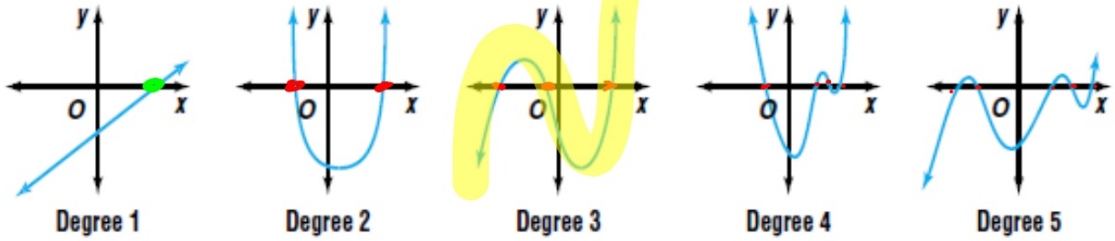
Roots (can be real or imag.) $x = 7$ $x = 2i$

imaginary number $\sqrt{-1}$

real number $5 + 0i$

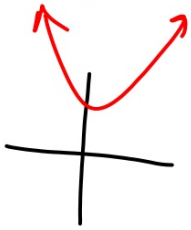
complex number $2 + 5i$ $0 + 7i$

Fundamental Theorem of Algebra number of sol = degree (at most)

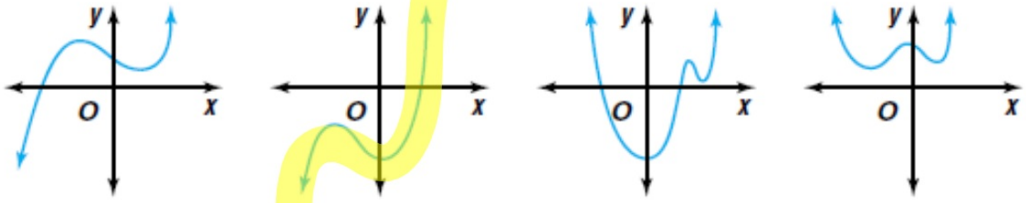


Degree 1 | Degree 2 | Degree 3 | Degree 4 | Degree 5

$y = x^1 - 6$ $y = x^2$ $y = x^3$



FTA: equation of degree n has at most n real solutions



Degree 3
1 *x*-intercept

Degree 3
1 *x*-intercept

Degree 4
2 *x*-intercepts

Degree 4
0 *x*-intercepts

State the degree and leading coefficient of each polynomial.

5. $a^3 + 6a + 14$

$d = 3$

$LC = 1$

6. $5m^2 + 8m^5 - 2$

$d = 5$

$LC = 8$

$8m^5 + 5m^2 - 2$

Determine whether each number is a root of $x^3 - 5x^2 - 3x - 18 = 0$. Explain.

7. 5

8. 6

$x = 5$ no

$5^3 - 5 \cdot 5^2 - 3 \cdot 5 - 18 \stackrel{?}{=} 0$

$125 - 125 - 15 - 18 \stackrel{?}{=} 0$

$-33 \neq 0$

Alg 2 4.3

Write a polynomial equation of least degree for each set of roots. Does the equation have an odd or even degree? How many times does the graph of the related function cross the x-axis?

9. $-5, 7$

10. $6, 2i, -2i, i, -i$

$x^2 - 2x - 35 = 0$

$(x+5)(x-7) = 0$

$x+5 = 0 \quad x-7 = 0$

$x = -5 \quad x = 7$

$x^2 + 5x + 6 = 0$

$(x+3)(x+2) = 0$

$x+3 = 0 \quad x+2 = 0$

$x = -3 \quad x = -2$

conjugate pair

$x = 2i \quad x = -2i$

$x = 3+5i \quad x = 3-5i$

$x^2 + 4 = 0$

$(x-2i)(x+2i) = 0$

$x-2i = 0 \quad x+2i = 0$

$x = 2i \quad x = -2i$

$x - 2i$

$x + 2i$

$x^2 + 2ix - 2ix - 4i^2$

$i^2 = -1$

$x^2 + 4$

$= 0$

$x^5 - 6x^4 + 5x^3 - 30x^2 + 4x - 24 = 0$

$(x-6)(x^4 + 5x^2 + 4) = 0$

$(x-6)(x^2+4)(x^2+1) = 0$

$(x-6)(x-2i)(x+2i)(x-i)(x+i) = 0$

$x = 6 \quad x = 2i \quad x = -2i \quad x = i \quad x = -i$

$x^2 + 4$

$x^2 + 1$

$x^4 + 5x^2 + 4$

$x - 6$

$-6x^4 - 30x^2 - 24$

$x^5 + 5x^3 + 4x$

$x = 6 \quad x = 3i \quad x = -3i$

$x^3 - 6x^2 + 9x - 54 = 0$

$(x-6)(x^2+9) = 0$

$(x-6)(x-3i)(x+3i) = 0$

$x - 6 = 0$

$x = 6$

$x - 3i = 0$

$x = 3i$

$x + 3i = 0$

$x = -3i$

$x^2 + 9$

$x - 6$

$-6x^2 - 54$

$x^3 \quad 9x$

$x - 3i$

$x + 3i$

$x^2 - 9i^2$

$i^2 = -1$

$x=3 \quad x=2-i$

$(x-3)(\quad)=0$

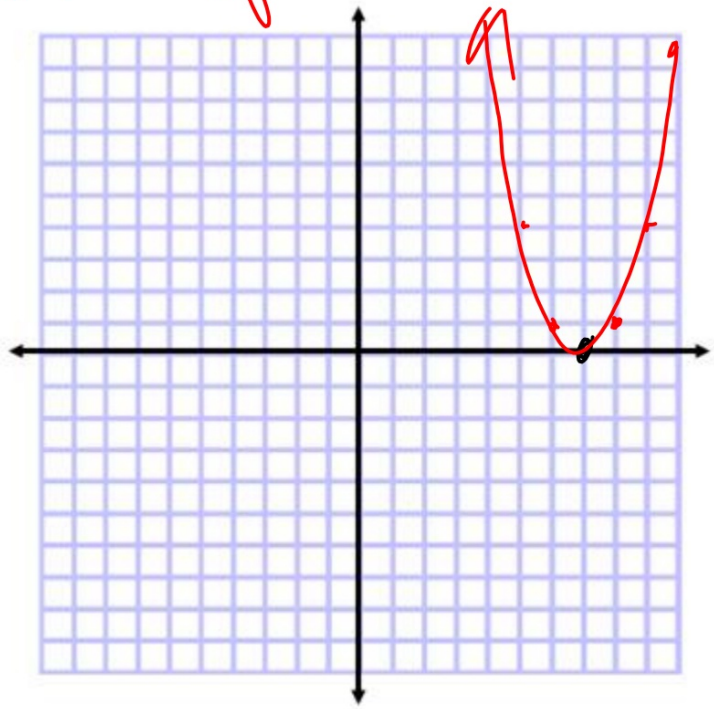
$(x)(x)(x-3)(x-2+i)(x-2-i)=0$

State the number of complex roots of each equation. Then find the roots and graph the related functions.

11. $x^2 - 14x + 49 = 0$ **12.** $a^3 + 2a^2 - 8a = 0$ **13.** $t^4 - 1 = 0$

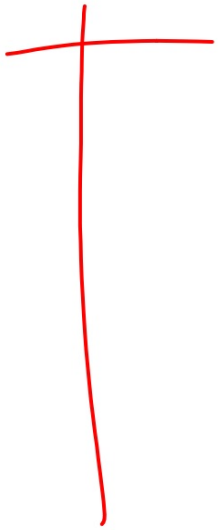
$y = x^2 - 14x + 49$

2 roots



Use everything that you know:
# of roots ✓
factoring ✓
crossing points ✓
end behavior

Show a (small) table of values

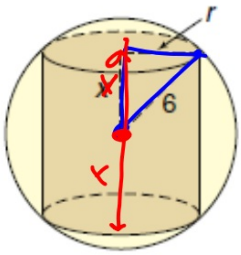


14. Geometry A cylinder is inscribed in a sphere with a radius of 6 units as shown.

a. Write a function that models the volume of the cylinder in terms of x. (*Hint:* The volume of a cylinder equals $\pi r^2 h$.)

b. Write this function as a polynomial function.

c. Find the volume of the cylinder if $x = 4$.



$V = \pi r^2 h$

$V = \pi\left(\sqrt{36-x^2}\right)^2 \cdot 2x$

$V = 2\pi x\left(36-x^2\right)$

$y = 72\pi x - 2\pi x^3$

$= 72\pi \cdot 4 - 2\pi \cdot 64$

$= 288\pi - 128\pi \quad = 160\pi$

$x^2 + r^2 = 36$

$-x^2 \qquad -x^2$

$r^2 = 36 - x^2$

$r = \pm\sqrt{36-x^2}$

It's messy, don't be scared.
Exact answer (in terms of π)

WB 4.1