

Trig 4.5

Approximate real zeros of a polynomial function
Use the upper and lower bound theorems

zero (of a function)

rational

real

Descarte's rule of signs

location principle

upper bound

lower bound

Use technology to help! But you still have to prove it!

graph...table

Approximate the real zeros of each function to the nearest tenth.

19. $f(x) = 3x^4 + x^2 - 1$

20. $f(x) = x^2 + 3x + 1$

Upper bound: There is only end behavior to the right of ...
(no max, min, zeros, etc.)
 $f(x)$...no sign changes in depressed polynomial

**Upper Bound
Theorem**

Suppose c is a positive real number and $P(x)$ is divided by $x - c$. If the resulting quotient and remainder have no change in sign, then $P(x)$ has no real zero greater than c . Thus, c is an upper bound of the zeros of $P(x)$.

Zero coefficients are ignored when counting sign changes.

Lower bound: There is only end behavior to the left of...
(no max, min, zeros, etc.)

$f(-c)$...no sign changes in depressed polynomial then c is a lower bound
(you try the opposite)

Goal: to zoom in as tightly as possible

A **lower bound** is an Integer less than or equal to the least real zero. A lower bound of the zeros of $P(x)$ can be found by determining an upper bound for the zeros of $P(-x)$.

| | |
|--------------------------------|--|
| Lower Bound Theorem | If c is an upper bound of the zeros of $P(-x)$, then $-c$ is a lower bound of the zeros of $P(x)$. |
|--------------------------------|--|

Use the Upper Bound Theorem to find an integral upper bound and the Lower Bound Theorem to find an integral lower bound of the zeros of each function.

26. $f(x) = 3x^3 - 2x^2 + 5x - 1$

27. $f(x) = x^2 - x - 1$

UB = 2

LB = -1

0?

$$\begin{array}{r} 2 \overline{) 1 \quad -1 \quad -1} \\ \underline{ 2 2} \\ 1 1 1 \end{array}$$

$f(-x) = x^2 + x - 1$

$$\begin{array}{r} 0 \overline{) 1 1 \quad -1} \\ \underline{ 0 0} \\ 1 1 -1 \end{array}$$

29. $f(x) = x^3 + 5x^2 - 3x - 20$

$$UB = 2$$

$$LB = -6$$

$$\begin{array}{r} 1 \overline{) 1 \quad 5 \quad -3 \quad -20} \\ \underline{ 1 \quad 6 \quad 3} \\ 1 \quad 6 \quad 3 \end{array}$$

$$f(-x) = -x^3 + 5x^2 + 3x - 20$$

$$\begin{array}{r} 6 \overline{) -1 \quad 5 \quad 3 \quad -20} \\ \underline{ -6 \quad -6 \quad -18} \\ -1 \quad -1 \quad -3 \quad -38 \end{array}$$

$$31. f(x) = x^5 + 5x^4 - 3x^3 + 20x^2 - 15$$

$$UB = 1$$

$$LB = -7$$

$$\begin{array}{r} 0 \rfloor \quad 1 \quad 5 \quad -3 \quad 20 \quad 0 \quad -15 \\ \quad \downarrow \quad 0 \quad 0 \quad 0 \end{array}$$

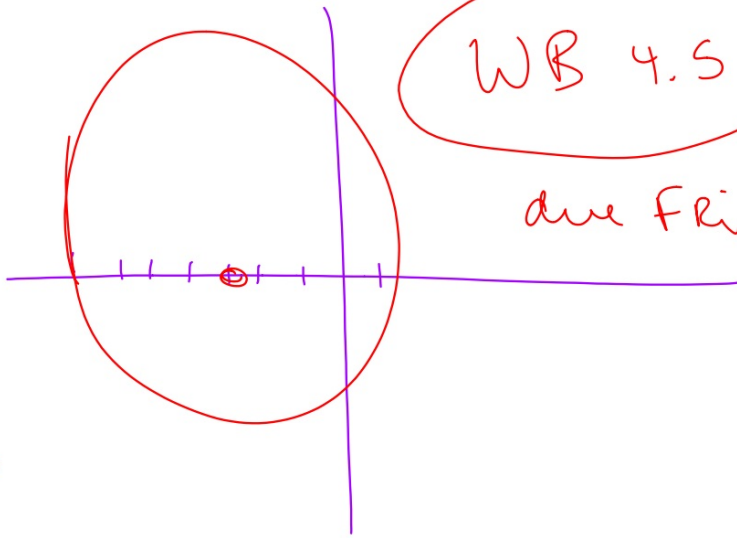
$$f(-x) \quad 1 \quad 5 \quad -3$$

$$-x^5 + 5x^4 + 3x^3 + 20x^2 - 15$$

$$\begin{array}{r} \uparrow \rfloor \quad -1 \quad 5 \quad 3 \quad 20 \quad 0 \quad -15 \\ \quad \downarrow \quad -7 \quad -14 \quad -77 \\ \hline -1 \quad -2 \quad -11 \quad -57 \end{array}$$

$$LB = -7$$

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WB 4.5

die Fri

-3,12