Trig 4.5

Approximate real zeros of a polynomial function

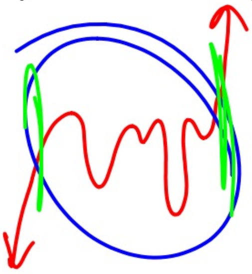
(could be rational, irrational)

Use the upper and lower bound theorems

no { x int, max/min

zero (of a function) = x-intercept

rational



integer (integral)

real

real: rational ($\sqrt{PS}$ = fraction form), irrat. ($\sqrt{nonPS}$)

not real: i

exact — roundoff or exact

Use everything that you know...

$2 \pm \sqrt{25}$

2 ± 5

(1) $3 \pm \sqrt{18}$

ex. $3 \pm 3\sqrt{2}$

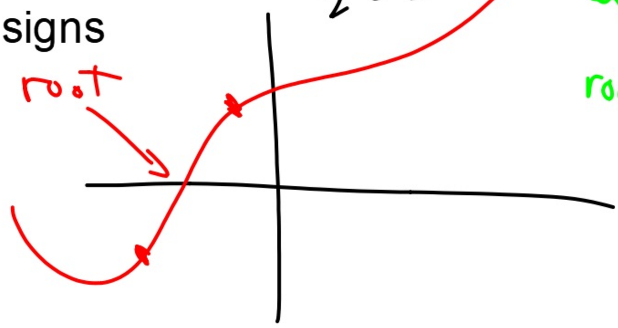
3 ± 4.24

round ≈ 7.24 or ≈ -1.24

9^2, 3^3

Descarte's rule of signs

location principle



upper bound

lower bound

1 **Determine between which consecutive integers the real zeros of $f(x) = x^3 - 4x^2 - 2x + 8$ are located.**

$-2 < x < -1$

$1 < x < 2$

$x = 4$

① 2, 0

② 1

~~imag 0, 2~~

$\begin{array}{c|cccc} & 1 & -4 & -2 & 8 \\ \hline \end{array}$

$x \approx -1$

$x \approx 1$

$x \approx 4$ 0

1. Graph and estimate (eyeball)
2. Table
3. You have to PROVE it.

2 Approximate the real zeros of $f(x) = 12x^3 - 19x^2 - x + 6$ to the nearest tenth.

$x \approx -0.5$
$x \approx 0.7$
graph then table
$x \approx 1.4$

opt.

pos 2, 0
neg 1
~~imag 0, 2~~

$12x^3 - 19x^2 - x + 6$

≈ -1
≈ 1
$1 < x < 2$

UB = 2
LB = -5

The **Upper Bound Theorem** will help you confirm whether you have determined all of the real zeros. An **upper bound** is an integer greater than or equal to the greatest real zero.

Upper Bound Theorem	Suppose c is a positive real number and $P(x)$ is divided by $x - c$. If the resulting quotient and remainder have no change in sign, then $P(x)$ has no real zero greater than c. Thus, c is an upper bound of the zeros of $P(x)$.

Zero coefficients are ignored when counting sign changes.

$$\begin{array}{r|rrrr} 4 & -1 & 3 & 5 & -10 \\ & \downarrow & -4 & -4 & \\ \hline & -1 & -1 & 1 & \end{array}$$

no sign change

3 Use the Upper Bound Theorem to find an integral upper bound and the Lower Bound Theorem to find an integral lower bound of the zeros of $f(x) = x^3 + 3x^2 - 5x - 10$.

$f(-x) = -x^3 + 3x^2 + 5x - 10$

Remember from Descarte's rule: sign changes are caused by zeros (x-intercepts=crossing points).

So if there are no more sign changes...

Try to be as specific as possible (narrow your window).

All real zeros will be found in the interval between LB and UB.
LB<all real zeros<UB

opposite/opposite... a little weird
The enemy of my enemy...

A **lower bound** is an integer less than or equal to the least real zero. A lower bound of the zeros of $P(x)$ can be found by determining an upper bound for the zeros of $P(-x)$.

Lower Bound Theorem	If c is an upper bound of the zeros of $P(-x)$, then $-c$ is a lower bound of the zeros of $P(x)$.

3 **Use the Upper Bound Theorem to find an integral upper bound and the Lower Bound Theorem to find an integral lower bound of the zeros of $f(x) = x^3 + 3x^2 - 5x - 10$.**

a little weird...
Use opposite:
f(-x) and -c

You can use technology to help, but you still have to prove it.
"Integral" refers to integer answers

Use the Upper Bound Theorem to find an integral upper bound and the Lower Bound Theorem to find an integral lower bound of the zeros of each function.

9. $f(x) = x^4 - 8x + 2$

10. $f(x) = x^4 + x^2 - 3$

Use the Upper Bound Theorem to find an integral upper bound and the Lower Bound Theorem to find an integral lower bound of the zeros of each function.

26. $f(x) = 3x^3 - 2x^2 + 5x - 1$

27. $f(x) = x^2 - x - 1$