Trig 8.4

2 $\perp$ vectors ?

Find the inner (dot) product of 2 vectors
Determine the cross product of 2 vectors
Determine whether 2 vectors are perpendicular

find 3rd $\perp$

product $\times$

perpendicular $m_1 \cdot m_2 = -1$

opp. & reciprocal

$\frac{2}{3} \cdot -\frac{3}{2} = \frac{-6}{6}$

$= -1$

dot product (inner product)

determinant

of a 2x2 matrix

of a 3x3 matrix

expansion by minors new

Cross product (vector)

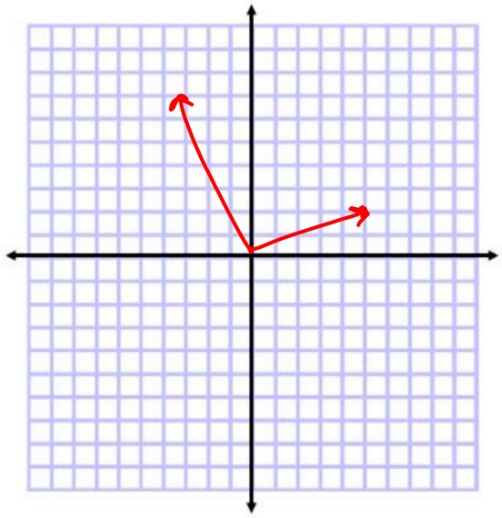
It's a slope thing...

$\langle a_1, a_2 \rangle$ $\langle b_1, b_2 \rangle$

$a_1b_1 + a_2b_2 = 0$

Ex. $\langle 3,5 \rangle$ and $\langle -5,3 \rangle$ reference triangles (slopes)

$-15 + 15 = 0$



Find each inner produc ular. Write
yes or *no.*

4. $\langle 5, 2 \rangle \cdot \langle -3, 7 \rangle = -1$ $3, 2, -2 \rangle$

$-15 + 14$

+ − + −
− + − +
+ − + −

3x3 determinant
(lesson 2.5)
cofactor method...

$$\begin{matrix} 1 & 3 & 0 \\ -1 & 2 & 4 \\ 6 & 0 & 5 \end{matrix}$$

$$+1\begin{vmatrix} 2 & 4 \\ 0 & 5 \end{vmatrix} - 3\begin{vmatrix} -1 & 4 \\ 6 & 5 \end{vmatrix} + 0\begin{vmatrix} -1 & 2 \\ 6 & 0 \end{vmatrix}$$

0, 10; 24, −5; 12, 0

$1 \cdot 10 + -3 \cdot -29 + 0 \cdot (-12)$

$10 + 87 + 0 = 97$

Did we do this? (asking...)

What would it look like for 3 lines to all be perpendicular to each other?

demo spaghetti
perpendicular to both?

+-+

Cross Product of Vectors in Space

If $\vec{\mathbf{a}} = \langle a_1, a_2, a_3 \rangle$ and $\vec{\mathbf{b}} = \langle b_1, b_2, b_3 \rangle$, then the cross product of $\vec{\mathbf{a}}$ and $\vec{\mathbf{b}}$ is defined as follows.

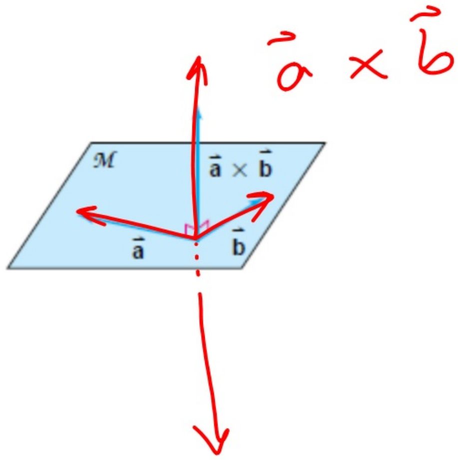
$$\vec{\mathbf{a}} \times \vec{\mathbf{b}} = +\begin{vmatrix} a_2 & a_3 \\ b_2 & b_3 \end{vmatrix} \vec{\mathbf{i}} - \begin{vmatrix} a_1 & a_3 \\ b_1 & b_3 \end{vmatrix} \vec{\mathbf{j}} + \begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix} \vec{\mathbf{k}}$$

$$\begin{matrix} \cancel{\vec{i}} & \cancel{\vec{j}} & \vec{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{matrix}$$

3x3 matrix...

i j k

also + - + etc.



cross prod is vector perp to both
could be above the plane or below the plane

$\vec{v} \times \vec{w} = \langle 5, 0, 0 \rangle$?

$\langle 0, 3, 1 \rangle$

$0 + 0 + 0 = 0$

$\overset{?}{=} 0$

$\langle 5, 0, 0 \rangle \overset{?}{=} 0$

$\langle 0, 1, 2 \rangle$

$0 + 0 + 0 = 0$

3 Find the cross product of $\vec{v}$ and $\vec{w}$ if $\vec{v} = \langle 0, 3, 1 \rangle$ and $\vec{w} = \langle 0, 1, 2 \rangle$. Verify that the resulting vector is perpendicular to $\vec{v}$ and $\vec{w}$.

$$\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & 3 & 1 \\ 0 & 1 & 2 \end{vmatrix} = +(\vec{i}) \begin{vmatrix} 3 & 1 \\ 1 & 2 \end{vmatrix} - (\vec{j}) \begin{vmatrix} 0 & 1 \\ 0 & 2 \end{vmatrix} + (\vec{k}) \begin{vmatrix} 0 & 3 \\ 0 & 1 \end{vmatrix}$$

$5\vec{i} - 0\vec{j} + 0\vec{k}$

$5\vec{i}$

$\vec{w} \times \vec{v} = ?$

Is $\bar{a} \times \bar{b} = \bar{b} \times \bar{a}$?

$\langle 13, 1, -5 \rangle$

$13 - 3 - 10 = 0$

Find each cross product. Then verify that the resulting vector is perpendicular to the given vectors.

$-26 + 1 + 25 = 0$

7. $\langle 1, -3, 2\rangle \times \langle -2, 1, -5\rangle$ $\quad$ **8.** $\langle 6, 2, 10\rangle \times \langle 4, 1, 9\rangle$

$\vec{a} \times \vec{b}$

$$\begin{array}{ccc} \vec{i} & \vec{j} & \vec{k} \\ 1 & -3 & 2 \\ -2 & 1 & -5 \end{array}$$

$-5 - -6$

$$+\vec{i}\begin{vmatrix} -3 & 2 \\ 1 & -5 \end{vmatrix} - \vec{j}\begin{vmatrix} 1 & 2 \\ -2 & -5 \end{vmatrix} + \vec{k}\begin{vmatrix} 1 & -3 \\ -2 & 1 \end{vmatrix}$$

Are their dot products = 0?

$\langle \quad \rangle + ans = 0$ $\quad 13\vec{i} + 1\vec{j} - 5\vec{k}$

$\langle \quad \rangle + ans = 0$

sketch… hint: it is a plane

9. Find a vector perpendicular to the plane containing the points (0, 1, 2), (−2, 2, 4), and (−1, −1, −1).

11-25 odd