Trig 7.4

$\times 2$

Use the double-angle identities for sine, cosine, tangent
Use the half-angle identities for sine, cosine, tangent

$\div 2 \quad \times \frac{1}{2}$

double

half

$2A \qquad \sin A\cos A + \cos A\sin A = 2\sin A\cos A$

$\sin(A \pm B) = \sin A\cos B \pm \cos A\sin B$

$2A \qquad \cos A\cos A - \sin A\sin A = \cos^2 A - \sin^2 A \longrightarrow$

$\cos(A+B) = \cos A\cos B \mp \sin A\sin B$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A\tan B} \longrightarrow$$

parking lot

$$\sin^2 A + \cos^2 A = 1$$
$$-\sin^2 A \qquad -\sin^2 A$$
$$\cos^2 A = 1 - \sin^2 A$$

~~sin(A + A)~~

cos(A + A) $= \cos^2 A - \sin^2 A$

$$= 1 - \sin^2 A - \sin^2 A$$
$$= 1 - 2\sin^2 A$$

$$\sin^2 + \cos^2 A = 1$$
$$-\cos^2 A \qquad -\cos^2 A$$
$$\sin^2 = 1 - \cos^2 A$$

$$\cos^2 A - (1 - \cos^2 A)$$
$$\cos^2 A - 1 + \cos^2 A$$
$$2\cos^2 A - 1$$

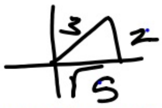
Double-Angle Identities	If θ represents the measure of an angle, then the following identities hold for all values of θ. $\sin 2\theta = 2 \sin \theta \cos \theta$ $\cos 2\theta = \cos^2 \theta - \sin^2 \theta$ $\cos 2\theta = 2 \cos^2 \theta - 1$ $\cos 2\theta = 1 - 2 \sin^2 \theta$ $\tan 2\theta = \frac{2 \tan \theta}{1 - \tan^2 \theta}$

$(2A)$

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B} = \frac{2 \tan A}{1 - \tan^2 A}$$

$x^2+4=9$

$x^2=5$



$\frac{2}{\sqrt{5}}$ $\quad \frac{2\sqrt{5}}{5}$

1 **If $\sin\theta = \frac{2}{3}$ and θ has its terminal side in the first quadrant, find the exact value of each function.**

Which quadrant?
Reference triangle?
Find all sides
Which identity?

a. $\sin 2\theta$ $= 2\sin\theta\cos\theta$

$= \frac{2}{1}\cdot\frac{2}{3}\cdot\frac{\sqrt{5}}{3}$

$= \frac{4\sqrt{5}}{9}$

b. $\cos 2\theta$ $= \cos^2 A - \sin^2 A$

$\frac{\sqrt{5}}{3}\cdot\frac{\sqrt{5}}{3} - \frac{2}{3}\cdot\frac{2}{3}$

$\frac{5}{9} - \frac{4}{9} = \frac{1}{9}$

c. $\tan 2\theta$ $= \frac{2\tan A}{1-\tan^2 A}$ $= \frac{\frac{2}{1}\left(\frac{2}{\sqrt{5}}\right)}{1-\left(\frac{2}{\sqrt{5}}\right)\left(\frac{2}{\sqrt{5}}\right)}$ $= \frac{\frac{4}{\sqrt{5}}}{1-\frac{4}{5}}$ $\quad \frac{\frac{4}{\sqrt{5}}}{\frac{1}{5}}$ $\quad \frac{5}{1}$

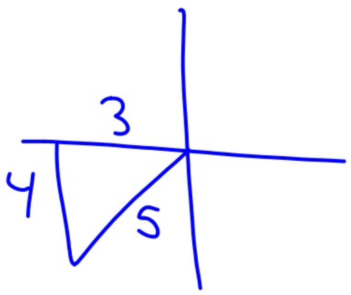
$\frac{20}{\sqrt{5}}$ $\quad \frac{20\sqrt{5}}{\sqrt{5}\sqrt{5}} = \frac{20\sqrt{5}}{5}$

$= 4\sqrt{5}$

Use the given information to find $\boxed{\sin 2\theta, \cos 2\theta, \text{ and } \tan 2\theta}$.

8. $\sin\theta = \frac{2}{5}$, $\boxed{0^\circ < \theta < 90^\circ}$ **9.** $\tan\theta = \frac{4}{3}$, $\boxed{\pi < \theta < \frac{3\pi}{2}}$

Which quadrant?
Reference triangle
Find all sides
Which identity?



9. $\sin 2\theta = \frac{24}{25}$

$\cos 2\theta = -\frac{7}{25}$

$\tan 2\theta = -\frac{24}{7}$

$$\frac{2 \cdot \frac{4}{3}}{1 - \left(\frac{4}{3} \cdot \frac{4}{3}\right)} \qquad \frac{\frac{8}{3}}{1 - \frac{16}{9}} = \frac{\frac{8}{3}}{-\frac{7}{9}} \qquad \frac{8}{3} \cdot -\frac{9}{7} = -\frac{72}{21}$$

$\cos 2\theta = 2\cos^2\theta - 1$

Solve for $\cos\theta$.

$+1$

$$\frac{\cos 2\theta + 1}{2} = \frac{2\cos^2\theta}{2}$$

$$\sqrt{\frac{\cos 2\theta + 1}{2}} = \sqrt{\cos^2\theta}$$

$$\pm\sqrt{\frac{\cos 2\theta + 1}{2}} = \cos\theta$$

$\cos 2\theta = 1 - 2\sin^2\theta$

Solve for $\sin\theta$.

$$\begin{matrix}(-1)\\(-1)\end{matrix}\ \frac{\cos 2\theta - 1}{-2} = \frac{-2\sin^2\theta}{-2}$$

$$\frac{1-\cos 2\theta}{2} = \sin^2\theta$$

$$\pm\sqrt{\frac{1-\cos 2\theta}{2}} = \sin\theta$$

x...2x

Half-Angle Identities	If α represents the measure of an angle, then the following identities hold for all values of α. $$\sin\frac{\alpha}{2} = \pm\sqrt{\frac{1-\cos\alpha}{2}}$$ $$\cos\frac{\alpha}{2} = \pm\sqrt{\frac{1+\cos\alpha}{2}}$$ $$\tan\frac{\alpha}{2} = \pm\sqrt{\frac{1-\cos\alpha}{1+\cos\alpha}},\ \cos\alpha \neq -1$$

Unlike with the double-angles identities, you must determine the sign.

+ or - from original quadrant

half/whole.....What is this angle half of? (should be a handy angle)
+ or - (from original question)

2 **Use a half-angle identity to find the exact value of each function.**

a. $\sin \frac{7\pi}{12}$

b. $\cos 67.5°$

half/whole

Use a half-angle identity to find the exact value of each function.

6. $\sin \frac{\pi}{8}$

7. $\tan 165^\circ$

4 Verify that $\frac{\cos 2\theta}{1 + \sin 2\theta} = \frac{\cot \theta - 1}{\cot \theta + 1}$ is an identity.

Both sides must match
Hey!

$\boxed{15-39 \text{ odd}}$

12. $y = 20\sin(3t + \theta)$ $\boxed{20(0 + 1\cdot\cos 3t)}$

$y = 20\sin(\underset{A}{3t} + \underset{B}{90}) = 20(\sin 3t \cos 90 + \cos 3t \sin 90)$

$y = 20\sin(\underset{A}{3t} + \underset{B}{270}) = 20(\sin 3t \cos 270 + \cos 3t \sin 270)$

$\boxed{20(0 - \cos 3t)}$

$20\cos 3t - 20\cos 3t$

$= 0$