Trig 7.3

Use sum and difference identities for sin, cos, tan

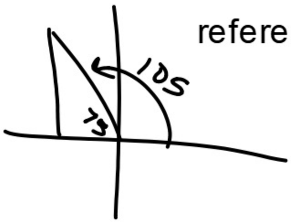
sum $\sin(A \pm B) =$

difference $\cos(A \pm B)$

verify (an identity) $( \quad ) = ( \quad )$

exact values 30, 45, 60 Q

reference triangles (if not in Quadrant-1)



| Sum and Difference Identities for the Cosine Function | If $\alpha$ and $\beta$ represent the measures of two angles, then the following identities hold for all values of $\alpha$ and $\beta$. $$\cos(\alpha \pm \beta) = \cos\alpha \cos\beta \mp \sin\alpha \sin\beta$$ |
|---|---|

| **Sum and Difference Identities for the Sine Function** | If $\alpha$ and $\beta$ represent the measures of two angles, then the following identities hold for all values of $\alpha$ and $\beta$. $$\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$$ |
|---|---|

**Use sum or difference identities to find the exact value of each trigonometric function.**

**5.** $\cos 165^\circ$ **6.** $\tan \frac{\pi}{12}$ **7.** $\sec 795^\circ$

$\left( \qquad \right)^{\text{recip}}$

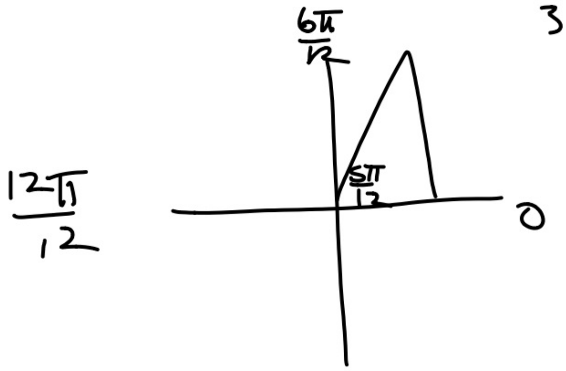
$$\frac{\sqrt{6}+\sqrt{2}}{4} \quad \frac{4}{\sqrt{6}+\sqrt{2}} \cdot \frac{\sqrt{6}-\sqrt{2}}{\sqrt{6}-\sqrt{2}}$$

$$\frac{4\sqrt{6}-4\sqrt{2}}{6-2} \quad \frac{\cancel{4}\sqrt{6}-\cancel{4}\sqrt{2}}{\cancel{4}} = \sqrt{6}-\sqrt{2}$$

Find the 1st quad. reference angle
Determine whether pos or neg answer

$\sin \frac{5\pi}{12}$ 75

$\frac{3}{3}$ $\frac{\pi}{4}$ $\frac{\pi}{3}$ $\frac{4}{4}$ $\frac{6\pi}{12}$




$\frac{4\pi - 3\pi}{12}$

$\frac{\pi}{4} + \frac{\pi}{6}$

$\frac{3\pi}{12} + \frac{2\pi}{12}$

| Sum and Difference Identities for the Tangent Function | If $\alpha$ and $\beta$ represent the measures of two angles, then the following identities hold for all values of $\alpha$ and $\beta$. $$\boxed{\tan(\alpha \pm \beta) = \frac{\tan\alpha \pm \tan\beta}{1 \mp \tan\alpha\tan\beta}}$$ |
|---|---|

*You will be asked to derive these identities in Exercise 47.*

Don't copy this all down, just watch
Will use but not derive the formula

$$\tan x = \frac{\sin x}{\cos x}$$

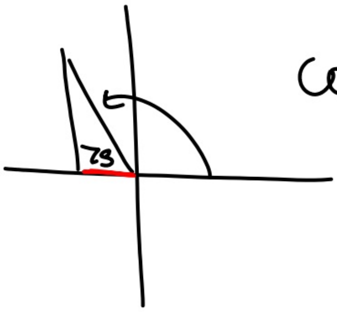
$$\tan(A \pm B) = \frac{\sin(A \pm B)}{\cos(A \pm B)} = \frac{\dfrac{\sin A \cos B}{\cos A \cdot \cos B} \pm \dfrac{\cos A \sin B}{\cos A \cos B}}{\dfrac{\cos A \cos B}{\cos A \cdot \cos B} + \dfrac{\sin A \sin B}{\cos A \cos B}}$$

$$= \frac{\tan A + \tan B}{1 - \tan A \cdot \tan B}$$

**14.** $\cos 105^\circ$ **15.** $\sin 165^\circ$

**17.** $\sin \frac{\pi}{12}$ **18.** $\tan 195^\circ$

pos or neg if outside Q1
recip if sec or csc




$$\cos 75 = \cos 30 \cdot \cos 45 - \sin 30 \cdot \sin 45$$

$$(\underset{A}{30} + \underset{B}{45}) = \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2} - \frac{1}{2} \cdot \frac{\sqrt{2}}{2}$$

$$= \frac{\sqrt{6} - \sqrt{2}}{4} \qquad -\left(\frac{\sqrt{6} - \sqrt{2}}{4}\right)$$

$$\tan(\overset{A}{45}-\overset{B}{30}) = \frac{\tan 45 - \tan 30}{1 + \tan 45 \cdot \tan 30}$$

$$= \frac{\left(\frac{3}{3}1 - \frac{\sqrt{3}}{3}\right)}{\frac{3}{3}\frac{1}{1} + 1 \cdot \frac{\sqrt{3}}{3}} \qquad \frac{\frac{3-\sqrt{3}}{\cancel{3}}}{\frac{3+\sqrt{3}}{3}} \qquad \frac{\cancel{3}}{3+\sqrt{3}} \qquad (2-\sqrt{3})$$

$$\frac{3-\sqrt{3}}{3+\sqrt{3}} \cdot \frac{3-\sqrt{3}}{3-\sqrt{3}}$$

$$= \frac{9-3\sqrt{3}-3\sqrt{3}+3}{9-3}$$

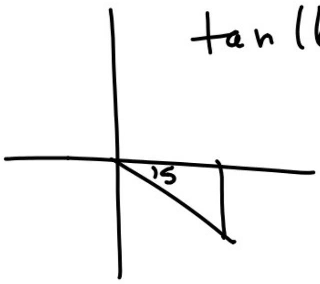
$$= \frac{12-6\sqrt{3}}{6}$$

$$\frac{12}{6} - \frac{6\sqrt{3}}{6}$$

$$2-\sqrt{3}$$

$\tan 345$

$$\tan(\overset{A}{60}-\overset{B}{45}) = \frac{\tan 60 - \tan 45}{1 + \tan 60 \tan 45}$$




$$= \frac{\sqrt{3}-1}{1+\sqrt{3}\cdot 1} = \frac{\sqrt{3}-1}{1+\sqrt{3}} \cdot \frac{1-\sqrt{3}}{1-\sqrt{3}} = -(2-\sqrt{3})$$

$$-2+\sqrt{3}$$

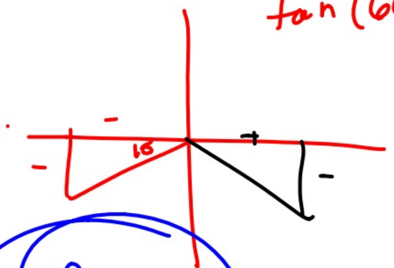
$$\frac{\sqrt{3}-3-1+\sqrt{3}}{1-\cancel{\sqrt{3}}+\cancel{\sqrt{3}}-3} = \frac{-4+2\sqrt{3}}{-2}$$

$$\frac{-4}{-2} + \frac{2\sqrt{3}}{-2}$$

$$2-\sqrt{3}$$

$\tan 195 = \tan 15$

$$\tan(\overset{A}{60} - \overset{B}{45}) = \frac{\tan 60 - \tan 45}{1 + \tan 60 \cdot \tan 45}$$




$$= \frac{\sqrt{3} - 1}{1 + \sqrt{3} \cdot 1}$$

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$$= \frac{\sqrt{3} - 1}{1 + \sqrt{3}} \cdot \frac{1 - \sqrt{3}}{1 - \sqrt{3}} \qquad +(2 - \sqrt{3})$$

$$= \frac{\sqrt{3} - 3 - 1 + \sqrt{3}}{1 - 3} = \frac{-4 + 2\sqrt{3}}{-2}$$

$$\frac{-4}{-2} + \frac{2\sqrt{3}}{-2}$$

$$2 - \sqrt{3}$$

$-(2 - \sqrt{2})$

$-2 + \sqrt{2}$

**5** **Use the sum or difference identity for tangent to find the exact value of $\tan 285°$.**

get both sides =
use appropriate + or - ident

**Verify that each equation is an identity.**

**10.** $\sin(90^\circ + A) = \cos A$

$\sin 90 \cos A + \cos 90 \sin A = \cos A$

$1 \cos A + 0 \cdot \sin A$

$\cos A = \cos A$

**6** **Verify that $\csc\left(\frac{3\pi}{2} + A\right) = -\sec A$ is an identity.**

$$\csc(270 + A) = -\sec A$$

$$\frac{1}{\sin 270 \cos A + \cos 270 \sin A} = \frac{-1}{\cos A}$$

$$\frac{1}{-1 \cos A + 0 \sin A} = \frac{-1}{\cos A}$$

$$\frac{1}{-\cos A} = \frac{-1}{\cos A}$$

FOIL

$$(x+3)(x-5)$$

$$x^2-5x+3x-15$$

$$x^2-2x-15$$