Trig 7.3

Use sum and difference identities for sin, cos, tan

sum $+$

difference $-$

verify (an identity) HEY!

special angles

exact values

Set your calc to degrees:

Is sin 75 = sin 30 + sin 45 ? no

Is cos 50 = cos 90 - cos 40 ? no

Is tan 23 = tan 73 - tan 50 ? no

You can use $\boxed{0\ 30\ 45\ 60\ 90.}$
(why?)
You can add or subtract

How can you make 15? = 45 - 30 = 60 - 45
75? = 30 + 45
105? = 60 + 45
120? = 90 + 30

Sum and Difference Identities for the Sine Function

If α and β represent the measures of two angles, then the following identities hold for all values of α and β.

$$\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$$

$$\sin 75 = \sin(\overset{A}{45}+\overset{B}{30}) = \sin 45 \cdot \cos 30 + \cos 45 \cdot \sin 30$$

$$= \left(\frac{\sqrt{2}}{2}\cdot\frac{\sqrt{3}}{2}\right) + \left(\frac{\sqrt{2}}{2}\cdot\frac{1}{2}\right)$$

$$= \frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4} = \frac{\sqrt{6}+\sqrt{2}}{4}$$

Sum and Difference Identities for the Cosine Function	If α and β represent the measures of two angles, then the following identities hold for all values of α and β. $$\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$$

0, 30, 45, 60, 90

Use sum and difference identities to find the exact value of cos75°

(how can we get 75?)

$$\cos(\overset{A}{30}+\overset{B}{45}) = \cos 30 \cos 45 - \sin 30 \sin 45$$

$$\left(\frac{\sqrt{3}}{2}\cdot\frac{\sqrt{2}}{2}\right) - \left(\frac{1}{2}\cdot\frac{\sqrt{2}}{2}\right)$$

$$\frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4} = \frac{\sqrt{6}-\sqrt{2}}{4}$$

2 **Use the sum or difference identity for cosine to find the exact value of cos 735°.**

How can I make 735?
Reference triangle
Special angle(s)
Use identity
Simplify
What quadrant? (pos or neg)

$\cos 15 =$

$\cos(\overset{A}{45} - \overset{B}{30}) = \cos 45 \cdot \cos 30 + \sin 45 \sin 30$

$= \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2} \cdot \frac{1}{2}$

$= \frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4} = -\left(\frac{\sqrt{6}+\sqrt{2}}{4}\right)$

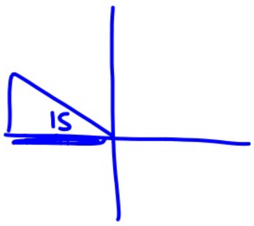
Can be radians or degrees

Use sum or difference identities to find the exact value of each trigonometric function.

5. $\cos 165^\circ$ =

6. $\tan \frac{\pi}{12}$ (15°)

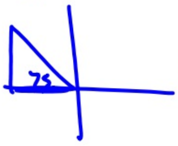
7. $\sec 795^\circ$ (75°)



14. $\cos 105°$ **15.** $\sin 165°$

17. $\sin \frac{\pi}{12}$ **18.** $\tan 195°$

(14)



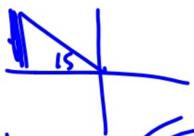
$\cos(45+30) = \cos 45 \cos 30 - \sin 45 \sin 30$

$= \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2} \cdot \frac{1}{2}$

$= \frac{\sqrt{6}-\sqrt{2}}{4}$

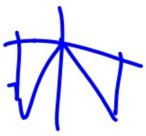
13-29 odds

(15)



$\sin(\overset{A}{45} - \overset{B}{30}) = \sin 45 \cos 30 - \cos 45 \sin 30$

$\frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2} \cdot \frac{1}{2}$



$\frac{\sqrt{6}-\sqrt{2}}{4}$

$\csc^2$

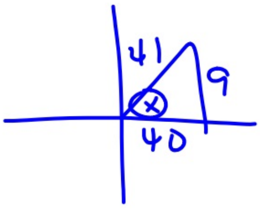
$$\frac{s^2}{s^2} + \frac{c^2}{s^2} = \frac{1}{s^2}$$

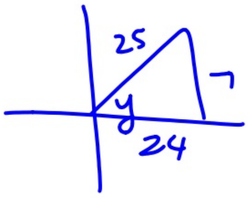
$$1 + \cot^2 = \csc^2$$

(A-B)

3 **Find the value of $\sin(x - y)$ if $0 < x < \frac{\pi}{2}$, $0 < y < \frac{\pi}{2}$, $\sin x = \frac{9}{41}$,** $\frac{o}{h}$ **and $\sin y = \frac{7}{25}$.** $\frac{o}{h}$

1. draw picture for x and y (what quadrant is it?)
2. analyze triangles
3. write trig identity
4. substitute & simplify





$$\sin(\overset{A}{x} - \overset{B}{y}) = \sin x \cos y - \cos x \sin y$$

$$= \frac{9}{41} \cdot \frac{24}{25} - \frac{40}{41} \cdot \frac{7}{25}$$

$$= \frac{216 - 280}{1025} = \boxed{\frac{-64}{1025}} =$$

Sum and Difference Identities for the Tangent Function	If α and β represent the measures of two angles, then the following identities hold for all values of α and β. $$\tan(\alpha \pm \beta) = \frac{\tan \alpha \pm \tan \beta}{1 \mp \tan \alpha \tan \beta}$$

You will be asked to derive these identities in Exercise 47.

5 **Use the sum or difference identity for tangent to find the exact value of $\tan 285°$.**

6 **Verify that** $\csc\left(\frac{3\pi}{2} + A\right) = -\sec A$ **is an identity.**

Verify that each equation is an identity.

10. $\sin(90^\circ + A) = \cos A$