

Trig 7.1

Identify and use trig identities

parking lot

identity $2(x+5) = 2x + 10$
 equation $x + 3 = 5$

counterexample

reciprocal $\frac{a}{b} \rightarrow \frac{b}{a}$

quotient $\div$

symmetry

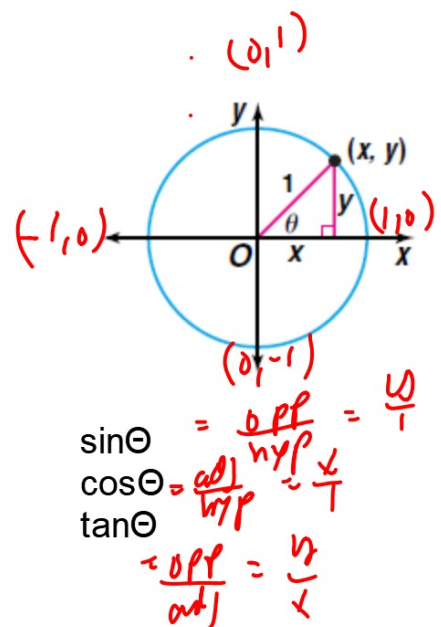
trig identities

pythagorean

opposite angle

parking lot

whiteboards (if time)

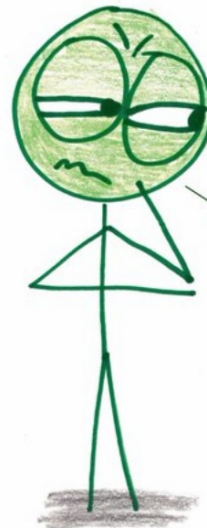


Identity: an equation that applies to all values



Anyone who tracks mud on my carpet will pay the price.

Conditional Equation: specifies values without necessarily naming them



Someone forgot to wipe their feet...

$$2(x+3) = 10$$

$$\begin{array}{r} 2x + 6 = 10 \\ -6 \quad -6 \\ \hline 2x = 4 \\ \frac{2}{2} \end{array}$$

$$x = 2$$

$$2(x+3) = 2x + 6$$

$$\begin{array}{r} 2x + 6 = 2x + 6 \\ -6 \quad \quad -6 \\ \hline 2x = 2x \\ -2x \quad -2x \\ \hline 0 = 0 \end{array}$$

Callback to geometry: All you need is **one** exception...

1 Prove that $\sin x \cos x = \tan x$ is *not* a trigonometric identity by producing a counterexample.

30°

$$\sin 30^\circ \cdot \cos 30^\circ = \tan 30^\circ$$

$$\frac{1}{2} \cdot \frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{3}$$

$$\frac{\sqrt{3}}{4} = \frac{\sqrt{3}}{3}$$

Definitions: Start parking lot

The following trigonometric identities hold for all values of θ where each expression is defined.

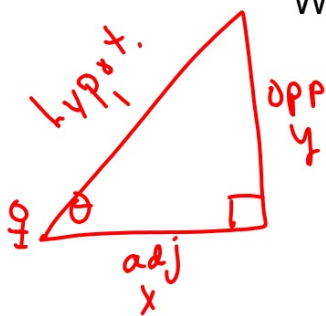
$$\begin{array}{ll} \sin \theta = \frac{1}{\csc \theta} & \cos \theta = \frac{1}{\sec \theta} \\ \csc \theta = \frac{1}{\sin \theta} & \sec \theta = \frac{1}{\cos \theta} \\ \tan \theta = \frac{1}{\cot \theta} & \cot \theta = \frac{1}{\tan \theta} \end{array}$$

It's not even Summer and I already
have a tan



The following trigonometric identities hold for all values of θ where each expression is defined.

$$\frac{\sin \theta}{\cos \theta} = \tan \theta \qquad \frac{\cos \theta}{\sin \theta} = \cot \theta$$



Why? (RT triangle)

$$\sin \theta = \frac{y}{x}$$

$$\cos \theta = \frac{x}{x}$$

$$\tan \theta = \frac{y}{x}$$

$$\tan \theta = \frac{\sin \theta}{\cos \theta}$$

Memorize (unit circle)

Pythagorean Identities

The following trigonometric identities hold for all values of θ where each expression is defined.

$$\sqrt{\sin^2 \theta + \cos^2 \theta} = \sqrt{1}$$

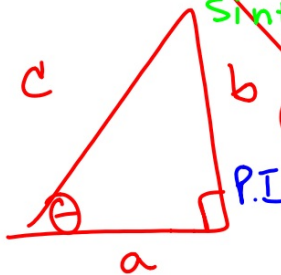
$$\tan^2 \theta + 1 = \sec^2 \theta$$

$$1 + \cot^2 \theta = \csc^2 \theta$$

$$\sin^2 \theta + \cos^2 \theta = 1$$

derive

derive



$$(\sin \theta)^2 + (\cos \theta)^2$$

$$\sin^2 \theta + \cos^2 \theta = 1$$

$$\frac{\sin^2 \theta + \cos^2 \theta}{\sin^2 \theta} = \frac{1}{\sin^2 \theta}$$

$$\begin{aligned} a^2 + b^2 &= c^2 \\ \frac{a^2}{c^2} + \frac{b^2}{c^2} &= \frac{c^2}{c^2} \\ \left(\frac{a}{c}\right)^2 + \left(\frac{b}{c}\right)^2 &= 1 \\ (\cos \theta)^2 + (\sin \theta)^2 &= 1 \end{aligned}$$

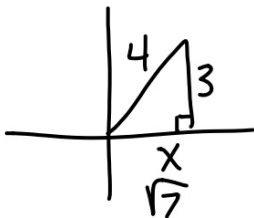
$$\left(\frac{\sin^2 \theta}{\cos^2 \theta} \right) + \frac{\cos^2 \theta}{\cos^2 \theta} = \frac{1}{\cos^2 \theta}$$

$$\sin^2 \theta + \cos^2 \theta = 1$$

Might be reciprocals
Might need reference triangle

2 Use the given information to find the trigonometric value.

a. If $\sec \theta = \frac{3}{2}$, find $\cos \theta = \frac{2}{3}$



$$\tan \theta = \frac{3}{\sqrt{7}} \cdot \frac{\sqrt{7}}{\sqrt{7}} = \frac{3\sqrt{7}}{7}$$

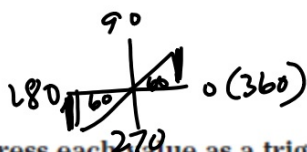
$$x^2 + 3^2 = 4^2$$

$$x^2 + 9 = 16$$

$$x^2 = 7$$

b. If $\csc \theta = \frac{4}{3}$, find $\tan \theta$.

hyp
opp



3 Express each value as a trigonometric function of an angle in Quadrant I.

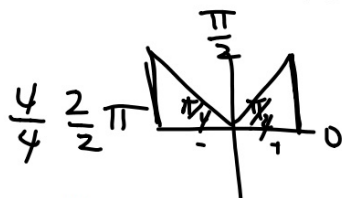
a. $\sin 600^\circ = -\sin 60^\circ$

$$\tan 600^\circ = \tan 60^\circ$$

(Compare to Quadrant I reference angle)

Uses same reference angle:

is the sign same or opposite?



$$\sin \frac{19\pi}{4} = \sin \frac{\pi}{4}$$

b. $\sin \frac{19\pi}{4} = \sin \frac{\pi}{4}$
 $\frac{8\pi}{4} = 2\pi$

c. $\cos(-410^\circ)$

d. $\tan \frac{37\pi}{6}$

**Opposite-
Angle
Identities**

The following trigonometric identities hold for all values of A .

$\sin A$
 $\cos A$

$$\sin(-A) = -\sin A$$

$$\cos(-A) = \cos A$$

19-430

CW
CCW

Same reference triangles but in
different quadrants: what has changed?
same or opposite?

What quadrant?
Sides of reference triangle
Answer the question

Use the given information to determine the exact trigonometric value.

8. $\cos \theta = \frac{2}{3}$, $0^\circ < \theta < 90^\circ$; $\sec \theta$

9. $\cot \theta = -\frac{\sqrt{5}}{2}$, $\frac{\pi}{2} < \theta < \pi$; $\tan \theta$

Express each value as a trigonometric function of an angle in Quadrant I.

12. $\cos \frac{7\pi}{3}$

13. $\csc (-330^\circ)$

same sign or opposite?

Try to get to a single trig function: what can you substitute?
(parking lot)

Simplify each expression.

14. $\frac{\csc \theta}{\cot \theta}$

4 Simplify $\sin x + \sin x \cot^2 x$.

Rules of algebra apply:
you can factor
combine like terms
you can substitute...
(parking lot)

15. $\cos x \csc x \tan x$

16. $\cos x \cot x + \sin x$