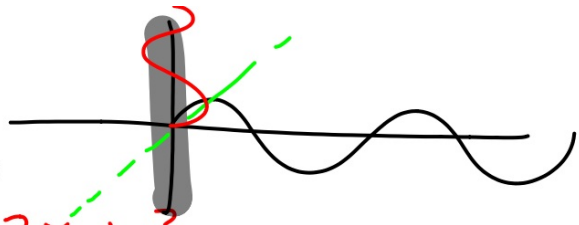
Trig 6.8

opposite thing

Graph inverse trig functions
Find principal values of inverse trig functions



$f(x)$ $f^{-1}(x)$

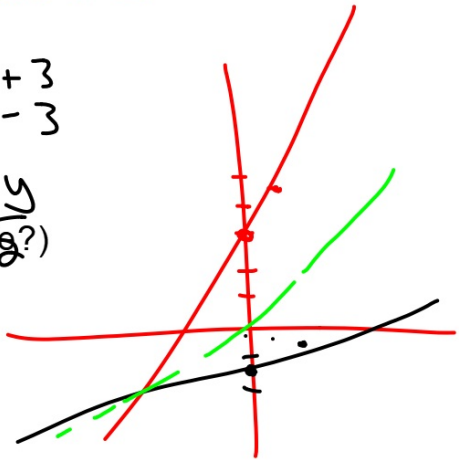
inverse (reflect over y=x)

$y = 2x + 3$

$x = 2y + 3$
$-3 \quad -3$

$\frac{x-3}{2} = \frac{2y}{2}$

$y = \frac{1}{2}x - \frac{3}{2}$



function
(sin cos etc....what is the ratio?)

inverse function
(inverse.sin inverse cos, etc....what is the angle?)

domain

range

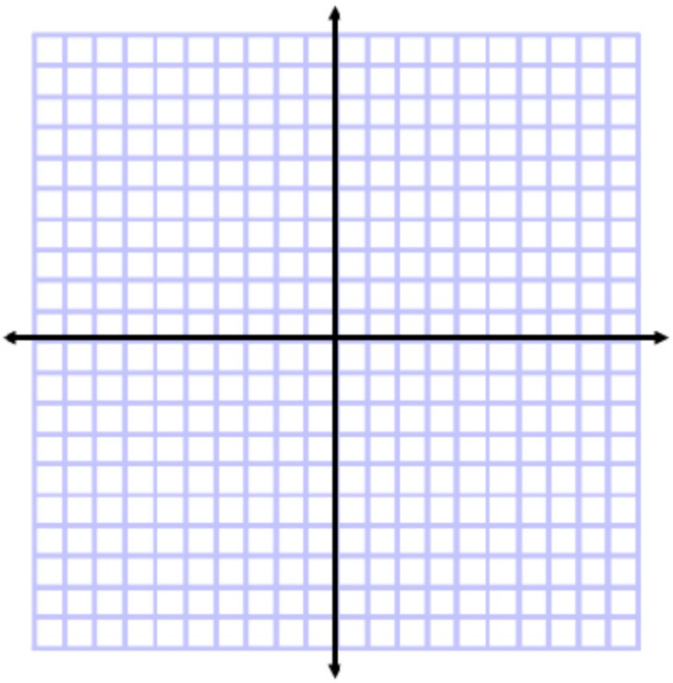
arcsin inverse sin $\sin^{-1}$
arccos inverse cos $\cos^{-1}$

principal values

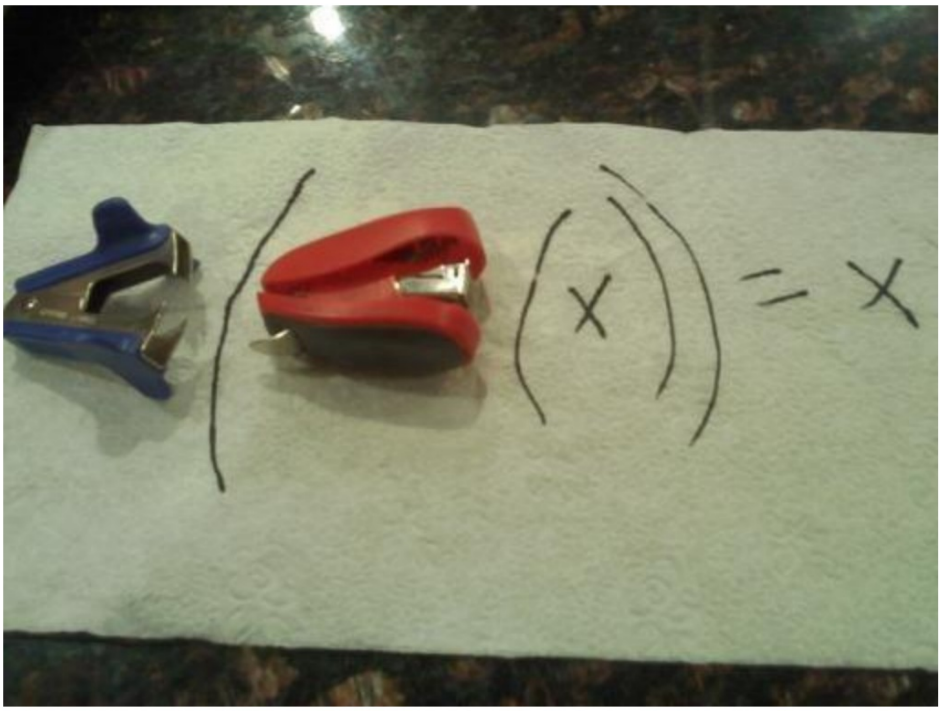
whiteboards

$y = 2x + 4$

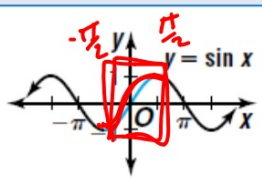
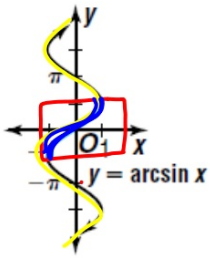
x&y trade places
reflect across y=x



inverse functions



Blue: One snip so that it can be a function

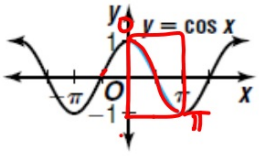
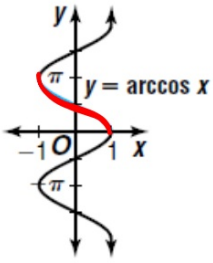
Relation	Ordered Pairs	Graph	Domain	Range
$y = \sin x$	$(x, \sin x)$		all real numbers	$-1 \le y \le 1$
$y = \arcsin x$ Sin⁻¹x sin⁻¹x Arcsin inverse sine	$(\sin x, x)$		$-1 \le x \le 1$	all real numbers

Inverse: x (domain) and y (range) trade places: reflect across y=x

$-\frac{\pi}{2} \longleftrightarrow \frac{\pi}{2}$

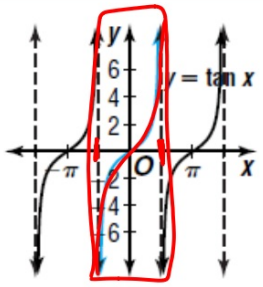
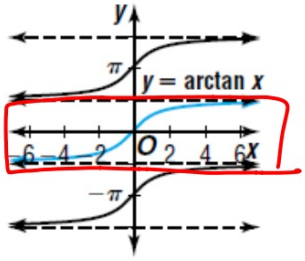
Is it a function?
Snip...

Blue: One snip so that it can be a function

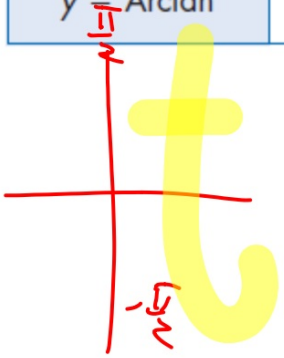
Relation	Ordered Pairs	Graph	Domain	Range
$y = \cos x$	$(x, \cos x)$		all real numbers	$-1 \le y \le 1$
$y = \arccos x$ Arccos $\cos^{-1}x$ $Cos^{-1}x$	$(\cos x, x)$		$-1 \le x \le 1$	all real numbers

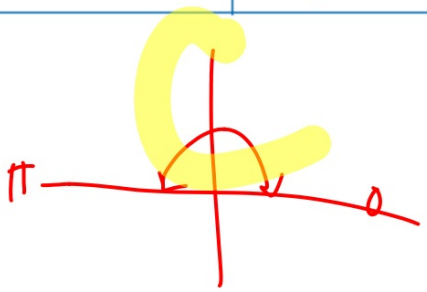
$0 \leftrightarrow \pi$

Is it a function?

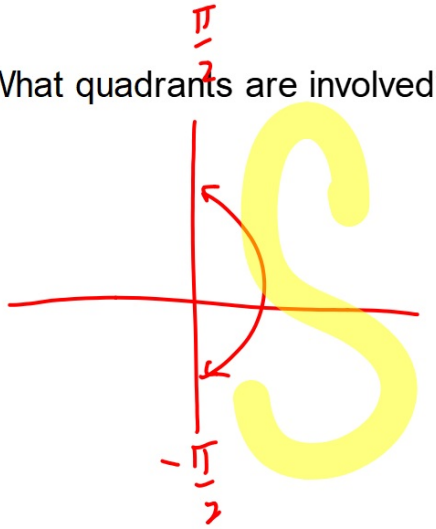
$y = \tan x$	$(x, \tan x)$		all real numbers except $\frac{\pi}{2}n$, where n is an odd integer	all real numbers
$y = \arctan x$	$(\tan x, x)$		all real numbers	all real numbers except $\frac{\pi}{2}n$, where n is an odd integer

Function	Domain	Range
$y = \text{Sin } x$ **	$-\frac{\pi}{2} \le x \le \frac{\pi}{2}$	$-1 \le y \le 1$
$y = \text{Arcsin } x$	$-1 \le x \le 1$	$-\frac{\pi}{2} \le y \le \frac{\pi}{2}$
$y = \text{Cos } x$	$0 \le x \le \pi$	$-1 \le y \le 1$
$y = \text{Arccos } x$	$-1 \le x \le 1$	$0 \le y \le \pi$
$y = \text{Tan } x$	$-\frac{\pi}{2} < x < \frac{\pi}{2}$	all real numbers
$y = \text{Arctan}$	all real numbers	$-\frac{\pi}{2} < y < \frac{\pi}{2}$





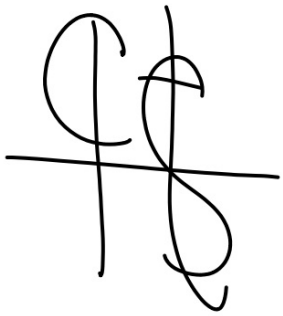
What quadrants are involved?



** Could be radians or degrees
Stupid Kroon trick...

Restricted quadrants by function

- (capital letter)
- "principal values"
- depends on which function



Arcsin=Invsin=$\sin^{-1}$

45°

$\frac{-\pi}{4}$

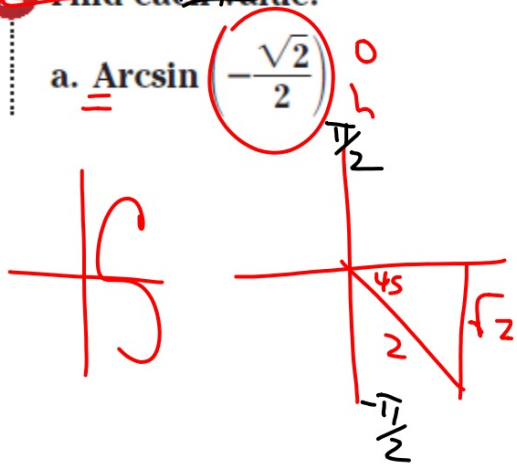
-45°

315°

$\frac{7\pi}{4}$

2 **Find each value.**

a. Arcsin $\left(-\frac{\sqrt{2}}{2}\right)$

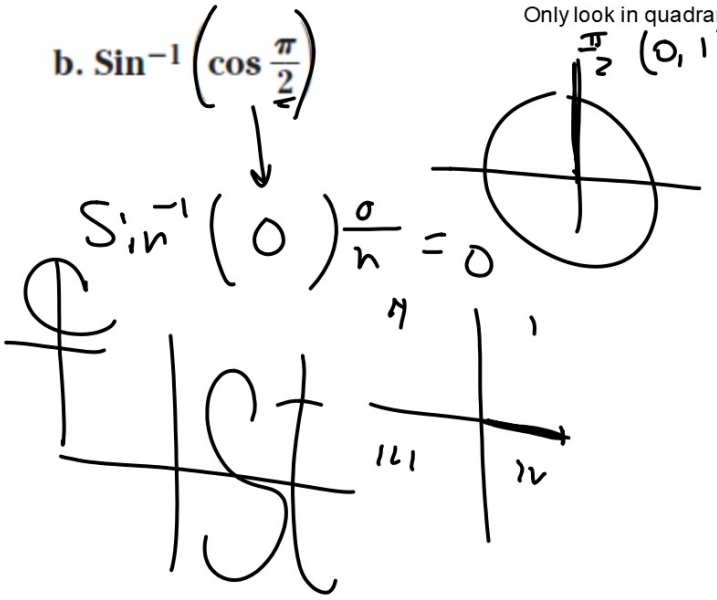


What angle has a sine of...
Only look in Quadrants 1 (if +) and 4 (if -)
Why?...

Parentheses first (just like always...)

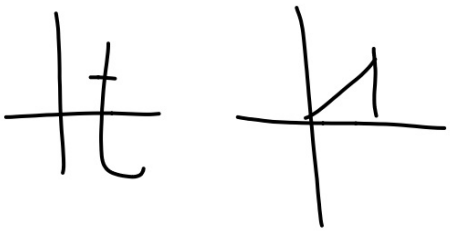
b. $\mathbf{Sin^{-1}}\left(\cos\frac{\pi}{2}\right)$

Only look in quadrants 1(+) and 4(-)



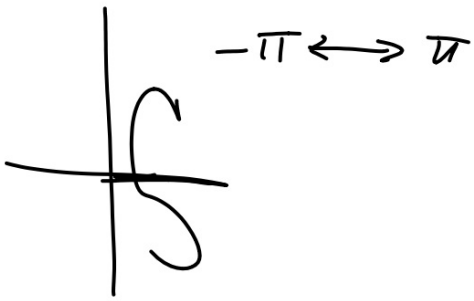
Find each value.

8. Arctan 1 $= \frac{\pi}{4}$

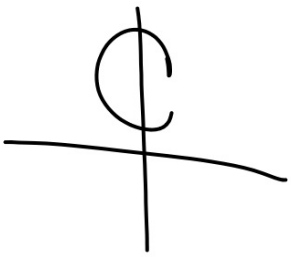


Quadrants:

22. $\sin^{-1} 0$ $\frac{0}{h} = 0$

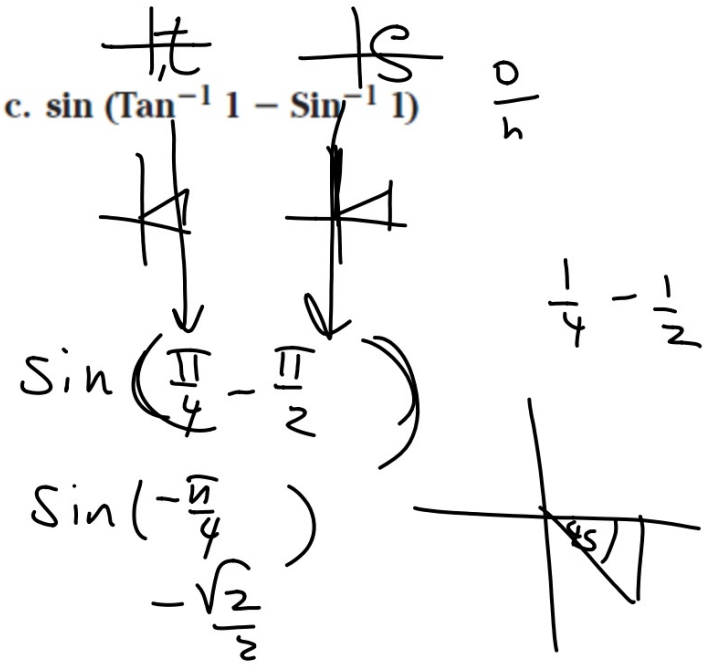


23. Arccos 0 $\frac{a}{h} = \frac{\pi}{2}$

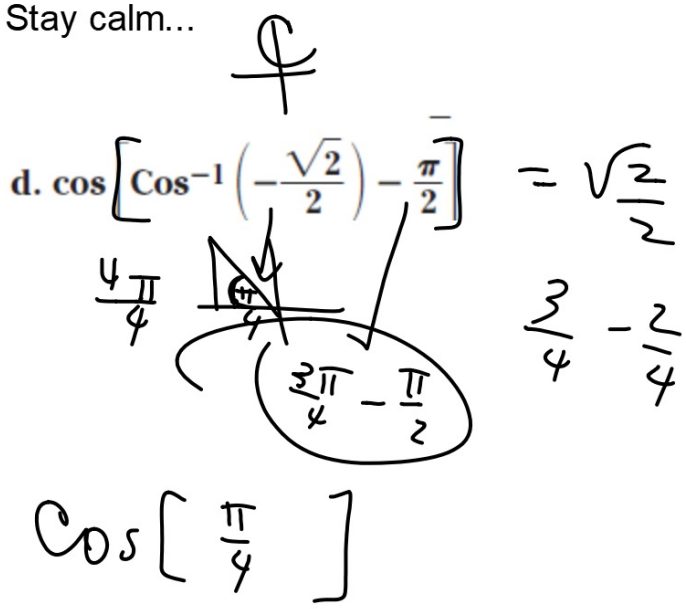


Quadrants:

c. $\sin(\text{Tan}^{-1} 1 - \text{Sin}^{-1} 1)$



Stay calm...


d. $\cos\left[\text{Cos}^{-1}\left(-\frac{\sqrt{2}}{2}\right)-\frac{\pi}{2}\right]$
$= \frac{\sqrt{2}}{2}$
$\frac{4\pi}{4}$
$\frac{\pi}{4}$
$\frac{3}{4}$ $-\frac{2}{4}$
$\frac{3\pi}{4}-\frac{\pi}{2}$
$\text{Cos}\left[\frac{\pi}{4}\right]$

9. $\cos\left(\text{Tan}^{-1}\,1\right)$

10. $\cos\left[\text{Cos}^{-1}\left(\frac{\sqrt{2}}{2}\right) - \frac{\pi}{2}\right]$

26. $\sin\left(2\,\mathrm{Cos}^{-1}\frac{\sqrt{2}}{2}\right)$

27. $\cos\left(\mathrm{Tan}^{-1}\sqrt{3}\right)$

28. $\cos\left(\text{Tan}^{-1}\,1 - \text{Sin}^{-1}\,1\right)$

29. $\cos\left(\text{Cos}^{-1}\,0 + \text{Sin}^{-1}\,\frac{1}{2}\right)$

tan (angle) = ratio → arctan (ratio) = angle
→ invtan
→ $\tan^{-1}$

$\tan = \frac{4}{3}$

21-37 ok