Trig 7.4

Use the double-angle identities for sine, cosine, tangent
Use the half-angle identities for sine, cosine, tangent

double

half

$\sin(A+A) = 2\sin A\cos A$

$\cos(A+A) = \cos^2 A - \sin^2 A$

$\tan(A+A) = \dfrac{2\tan A}{1-\tan^2 A}$

parking lot

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|---|---|
| **Double-Angle Identities** | If $\theta$ represents the measure of an angle, then the following identities hold for all values of $\theta$.<br>$\sin 2\theta = 2 \sin \theta \cos \theta$<br>$\cos 2\theta = \cos^2 \theta - \sin^2 \theta$<br>$\cos 2\theta = 2 \cos^2 \theta - 1$<br>$\cos 2\theta = 1 - 2 \sin^2 \theta$<br>$\tan 2\theta = \frac{2 \tan \theta}{1 - \tan^2 \theta}$ |

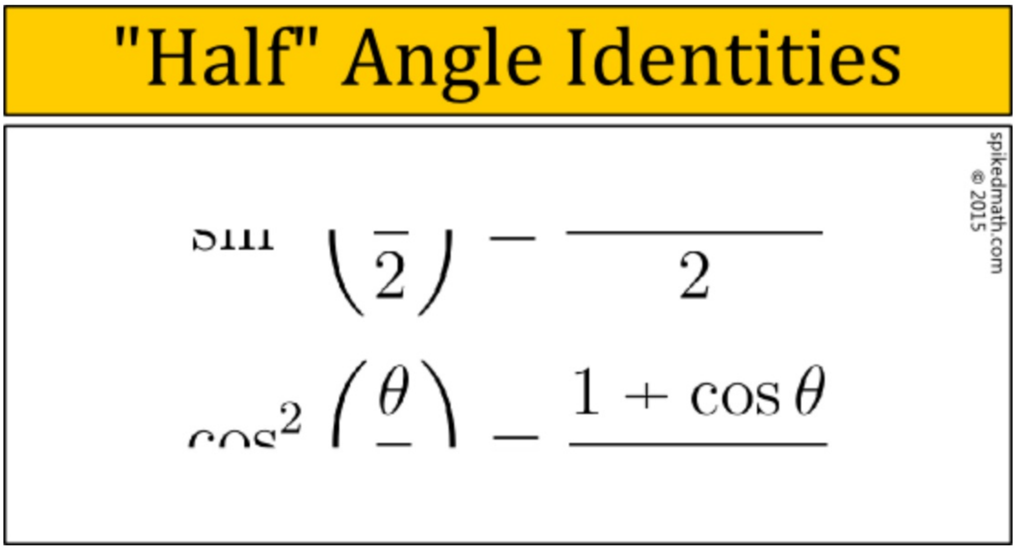
**Use the given information to find sin $2\theta$, cos $2\theta$, and tan $2\theta$.**

**21.** $\cos\theta = \frac{4}{5}, 0^\circ < \theta < 90^\circ$

**22.** $\sin\theta = \frac{1}{3}, 0 < \theta < \frac{\pi}{2}$

draw picture
analyze triangle
choose identity
substitute & solve

Next: half angle identities


"Half" Angle Identities
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|---|---|
| **Half-Angle Identities** | If $\alpha$ represents the measure of an angle, then the following identities hold for all values of $\alpha$.<br>$\sin \frac{\alpha}{2} = \pm\sqrt{\frac{1 - \cos \alpha}{2}}$<br>$\cos \frac{\alpha}{2} = \pm\sqrt{\frac{1 + \cos \alpha}{2}}$<br>$\tan \frac{\alpha}{2} = \pm\sqrt{\frac{1 - \cos \alpha}{1 + \cos \alpha}}, \cos \alpha \neq -1$ |

*Unlike with the double-angles identities, you must determine the sign.*

What quadrant is original question in? (+ or -)

What is it half of?
What quadrant is it in?
Which formula to use?

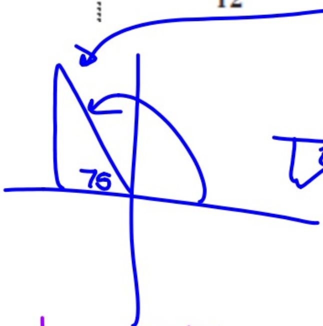

$$\pm\sqrt{\frac{1-\cos 45}{2}} = +\sqrt{\frac{\left(\frac{2}{2}-\frac{\sqrt{2}}{2}\right)}{2}}$$

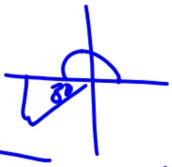
b. sin 22.5

$$= +\sqrt{\frac{\frac{2-\sqrt{2}}{2}}{2}}\cdot\frac{1}{2} = \sqrt{\frac{2-\sqrt{2}}{4}} = \frac{+\sqrt{2-\sqrt{2}}}{2}$$

**2** **Use a half-angle identity to find the exact value of each function.**

**a.** $\sin\frac{7\pi}{12}$ sin105




$$\pm\sqrt{\frac{1-\cos 210}{2}} = +\sqrt{\frac{\frac{2}{2}+\frac{\sqrt{3}}{2}}{2}}$$

$$+\sqrt{\frac{\frac{2+\sqrt{3}}{2}}{2}}\cdot\frac{1}{2} = +\sqrt{\frac{2+\sqrt{3}}{4}} = \frac{+\sqrt{2+\sqrt{3}}}{2}$$

$$\sqrt{\frac{(4+\sqrt{3})}{4}} = \frac{\sqrt{4+\sqrt{3}}}{2}$$

b. cos 67.5°

11) $\tan\frac{\pi}{8}$

$\tan 22.5$

$$\pm\sqrt{\frac{1-\cos 45}{1+\cos 45}} = \pm\sqrt{\frac{\cancel{\frac{2}{2}}-\frac{\sqrt{2}}{2}}{\cancel{\frac{2}{2}}+\frac{\sqrt{2}}{2}}} =$$

$$\pm\sqrt{\frac{\frac{2-\sqrt{2}}{\cancel{2}}}{\frac{2+\sqrt{2}}{2}}} \quad \frac{\cancel{2}}{2+\sqrt{2}} = \pm\sqrt{\frac{2-\sqrt{2}}{2+\sqrt{2}}\cdot\frac{2-\sqrt{2}}{2-\sqrt{2}}}$$

$$= \pm\sqrt{\frac{4-4\sqrt{2}+2}{4-2}} = \pm\sqrt{\frac{6-4\sqrt{2}}{2}} = \pm\sqrt{\frac{\cancel{2}(3-2\sqrt{2})}{\cancel{2}}} \quad \pm\sqrt{3-2\sqrt{2}}$$

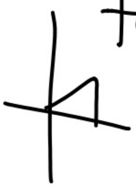
Use a half-angle identity to find the exact value of each function.

6. $\sin \frac{\pi}{8}$

7. $\tan 165°$

$$19 = \pm\sqrt{\frac{1-\cos 45}{1+\cos 45}} = \pm\sqrt{\frac{\frac{2}{2}-\frac{\sqrt{2}}{2}}{\frac{2}{2}+\frac{\sqrt{2}}{2}}}$$

$\tan 22.5$



$$= +\sqrt{\frac{\frac{2-\sqrt{2}}{\cancel{2}}}{\frac{2+\sqrt{2}}{2}} \cdot \frac{\cancel{2}}{2+\sqrt{2}}} = +\sqrt{\frac{2-\sqrt{2}}{2+\sqrt{2}}}$$

**Use a half-angle identity to find the exact value of each function.**

**14.** $\cos 15^\circ$

**15.** $\sin 75^\circ$

**16.** $\tan \frac{5\pi}{12}$