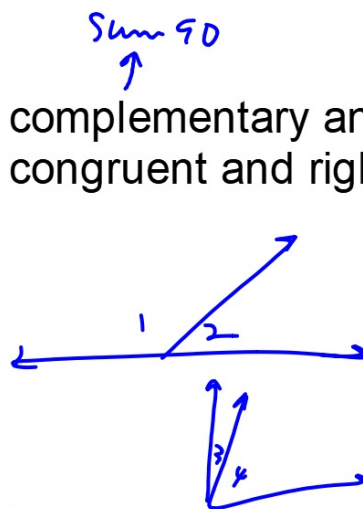Geometry 2.8
Write proofs involving complementary and supplementary angles
Write proofs involving congruent and right angles

Sum 90

Sum 180

complementary
supplementary
linear pair (Supp)
adjacent angles
protractor postulate
angle addition postulate

scrambled proofs
activity: plates, stirring sticks, circles
        scrambled proofs

# 1 Supplementary and Complementary Angles  The Protractor Postulate illustrates the relationship between angle measures and real numbers.

## Postulate 2.10  Protractor Postulate

**Words**  Given any angle, the measure can be put into one-to-one correspondence with real numbers between 0 and 180.
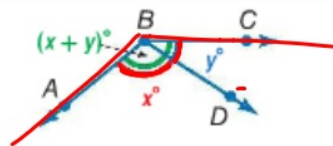
**Example**  If $\overrightarrow{BA}$ is placed along the protractor at 0°, then the measure of $\angle ABC$ corresponds to a positive real number.
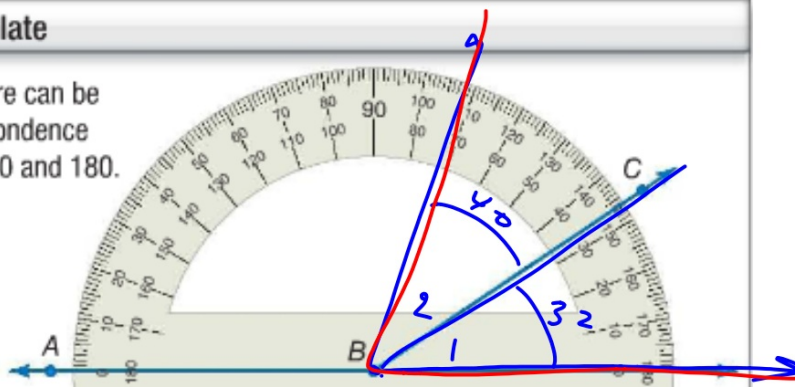
One small angle + other small angle = whole thing
(The ray inside doesn't take up any of the angle.)

$D$ is in the interior of $\angle ABC$ if and only if
$m\angle ABD + m\angle DBC = m\angle ABC.$

$(x + y)°$  B   C
A   $y°$
$x°$   D

**Example 1** Use the Angle Addition Postulate

Find $m\angle 1$ if $m\angle 2 = 56$ and $m\angle JKL = 145$.

$m\angle 1 = ?$ 89°

$145 = 56 + ?$
$-56 \quad -56$

J   K

J   K
2   1
$56$   ?   L
$145$

**Guided**Practice

1. If $m\angle 1 = 23$ and $m\angle ABC = 131$, find the measure
   of $\angle 3$. Justify each step.

$\angle 1 + \angle 2 + \angle 3 = 131$   angle add

$23 + 90 + \angle 3 = 131$   subs

$113 + \angle 3 = 131$   subs
$-113 \qquad -113$

$\angle 3 = 18°$   Subtr.

$131$
$23$   2   3
A   ?
B   C
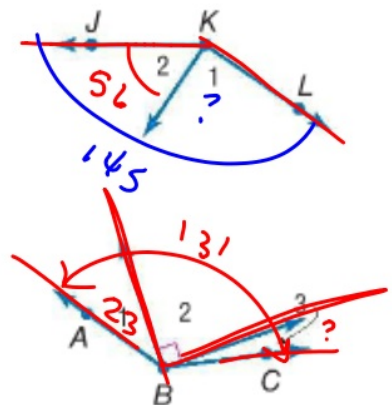
plates

*p. 152*

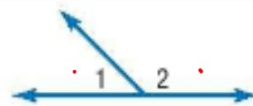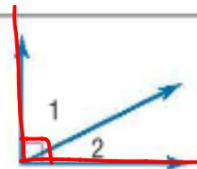## Theorems

*def L. P.*

**2.3** ~~Supplement Theorem~~ If two angles form a linear pair, then they are supplementary angles.

**Example** $m\angle 1 + m\angle 2 = 180$

*def comp*

**2.4** ~~Complement Theorem~~ If the noncommon sides of two adjacent angles form a right angle, then the angles are complementary angles.
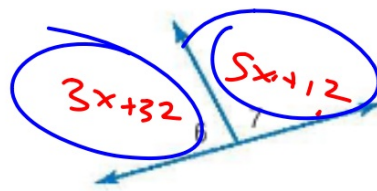
**Example** $m\angle 1 + m\angle 2 = 90$

You will prove Theorems 2.3 and 2.4 in Exercises 16 and 17, respectively.

**2.** ∠6 and ∠7 form linear pair. If $m\angle 6 = 3x + 32$
and $m\angle 7 = 5x + 12$, find $x$, $m\angle 6$, and $m\angle 7$.
Justify each step.

$3x+32$   $5x+12$

$3x+32 + 5x+12 = 180$    def ∠ P       $x = 17$
$8x + 44 = 180$           Subs          $m\angle 6 = 83$
$\quad -44 \quad -44$      Subtr.        $m\angle 7 = 97$
$\dfrac{8x}{8} = \dfrac{136}{8}$

$x = 17$,                 div.

**2** **Congruent Angles** The properties of algebra that applied to the congruence of segments and the equality of their measures also hold true for the congruence of angles and the equality of their measures.

*p. 153*

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**Theorem** 2.5  **Properties of Angle Congruence**

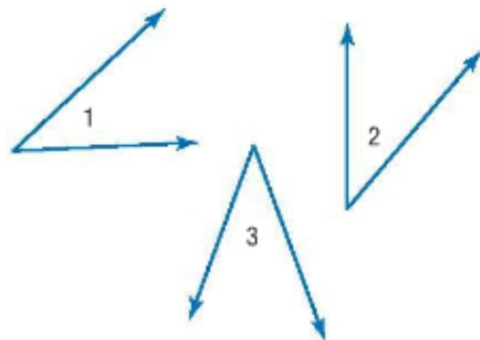**Reflexive Property of Congruence**

$\angle 1 \cong \angle 1$

**Symmetric Property of Congruence**

If $\angle 1 \cong \angle 2$, then $\angle 2 \cong \angle 1$.

**Transitive Property of Congruence**

If $\angle 1 \cong \angle 2$ and $\angle 2 \cong \angle 3$, then $\angle 1 \cong \angle 3$.

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You will prove the Reflexive and Transitive Properties of Congruence in Exercises 18 and 19, respectively.

**Proof** One Case of the Congruent Supplements Theorem

**Given:** $\angle 1$ and $\angle 2$ are supplementary.
$\angle 2$ and $\angle 3$ are supplementary.

**Prove:** $\angle 1 \cong \angle 3$

**Proof:**

| Statements | Reasons |
|---|---|
| 1. $\angle 1$ and $\angle 2$ are supplementary. | 1. Given |



$$\angle 1 + \angle 2 = 180$$
$$\angle 2 + \angle 3 = 180$$

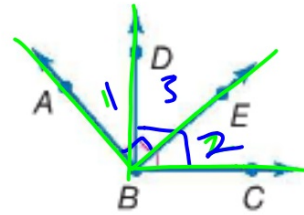$$\angle 1 + \angle 2 = \angle 2 + \angle 3$$
$$\underline{\quad -\angle 2 \quad -\angle 2 \quad}$$
$$\angle 1 = \angle 3$$
$$\angle 1 \cong \angle 3$$

**GuidedPractice**

**3.** In the figure, ∠ABE and ∠DBC are right angles.
Prove that ∠ABD ≅ ∠EBC.



| | | |
|---|---|---|
| 1. | ∠ABE + ∠DBC are rt ∠s | 1. given |
| 2. | ∠1 + ∠3 = 90 | 2. ∠ add |
| | ∠3 + ∠2 = 90 | |
| 3 | ∠1 + ∠3 = ∠3 + ∠2 | 3. Subs |
| 4. | ∠1 ≅ ∠2 | 4. Subtr |

G: term
P: term
given

Reminder: Required for proofs
You MUST include the given, prove, and drawing...
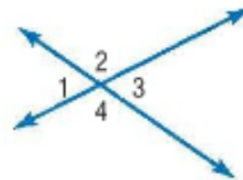if you want credit...

stirring sticks

2.8    9-~~13~~ 14 all
        37-51 o

**Theorem** 2.8  **Vertical Angles Theorem**

If two angles are vertical angles, then they are congruent.

**Abbreviation**  *Vert. ⦟ are ≅.*

**Example**    ∠1 ≅ ∠3 and ∠2 ≅ ∠4

You will prove Theorem 2.8 in Exercise 28.

You trusted me (so far) but now we are going to PROVE it!
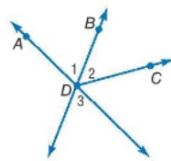
**Example 4** Use Vertical Angles

Prove that if $\overrightarrow{DB}$ bisects $\angle ADC$, then $\angle 2 \cong \angle 3$.

**Given:** $\overrightarrow{DB}$ bisects $\angle ADC$.

**Prove:** $\angle 2 \cong \angle 3$

**Proof:**

| Statements | Reasons |
|---|---|
| **1.** $\overrightarrow{DB}$ bisects $\angle ADC$. | **1.** Given |

What does "bisect" tell us?