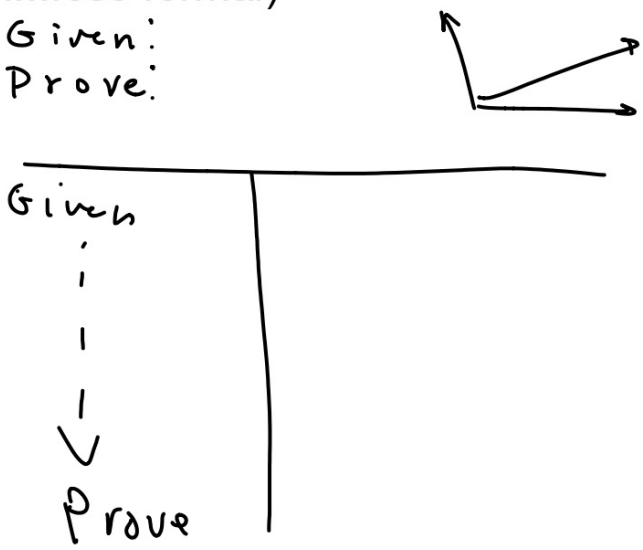
Geometry 2.6
Use algebra to write 2-column proofs
Use properties of equality to write geometric proofs

algebraic proof
paragraph proof (sentence form...less formal)
2-column proof (formal proof)

Given (hypothesis)
Prove (conclusion)
Picture (if any)
Reasoning from G to P

activity: scrambled proofs



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KeyConcept Properties of Real Numbers

The following properties are true for any real numbers a, b, and c.

Addition Property of Equality	If $a = b$, then $a + c = b + c$.
Subtraction Property of Equality	If $a = b$, then $a - c = b - c$.
Multiplication Property of Equality	If $a = b$, then $a \cdot c = b \cdot c$.
Division Property of Equality	If $a = b$ and $c \neq 0$, then, $\frac{a}{c} = \frac{b}{c}$.
Reflexive Property of Equality	$a = a$
Symmetric Property of Equality	If $a = b$, then $b = a$.
Transitive Property of Equality	If $a = b$ and $b = c$, then $a = c$.
Substitution Property of Equality	If $a = b$, then a may be replaced by b in any equation or expression.
Distributive Property	$a(b + c) = ab + ac$

scrambled proofs

An **algebraic proof** is a proof that is made up of a series of algebraic statements. The properties of equality provide justification for many statements in algebraic proofs.

given prove

Example 1 Justify Each Step When Solving an Equation

Prove that if $-5(x + 4) = 70$, then $x = -18$. Write a justification for each step.

$-5(x + 4) = 70$	Original equation or Given
-5	

Given: $-5(x+4)=70$
Prove: $x=-18$

Statement	Reason
1. $-5(x+4)=70$	1. Given
2. $-5x+-20=70$ $+20 \quad +20$	2. distrib. prop.
3. $\frac{-5x}{-5}=\frac{90}{-5}$	3. addition
4. $x=-18$	4. division

Real-World Example 2 Write an Algebraic Proof

SCIENCE If the formula to convert a Fahrenheit temperature to a Celsius temperature is $C = \frac{5}{9}(F - 32)$, then the formula to convert a Celsius temperature to a Fahrenheit temperature is $F = \frac{9}{5}C + 32$. Write a two-column proof to verify this conjecture.

given

prove

scrambled proofs

Begin by stating what is given and what you are to prove.

Given: $C = \frac{5}{9}(F - 32)$

Prove: $F = \frac{9}{5}C + 32$

Where do I start?
Where am I trying to go?

Proof:

1. $C = \frac{5}{9}(F - 32)$	**1.** Given

2.

3.

4.

5.

6.

Given: $C = \frac{5}{9}(F-32)$

Prove: $F = \frac{9}{5}C + 32$

	St.	Reas.
1.	$C = \frac{5}{9}(F-32)$	1. given
2.	$C = \frac{5}{9}F - 17\frac{7}{9}$ $-C \quad\quad -C$ $-\frac{5}{9}F \quad -\frac{5}{9}F$	2. distr. prop
3.	$-\frac{9}{5}(-\frac{5}{9})F = -\frac{9}{5}(-C) - \frac{9}{5}(17\frac{7}{9})$	3. add.
4.	$F = \frac{9}{5}C + 32$	4. mult

StudyTip

Commutative and Associative Properties

Throughout this text we shall assume that if a, b, and c are real numbers, then the following properties are true.

Commutative Property of Addition

$a + b = b + a$

Commutative Property of Multiplication

$a \cdot b = b \cdot a$

Associative Property of Addition

$(a + b) + c = a + (b + c)$

Associative Property of Multiplication

$(a \cdot b) \cdot c = a \cdot (b \cdot c)$

Callback to algebra:

Property	Segments	Angles
Reflexive	$AB = AB$	$m\angle 1 = m\angle 1$
Symmetric	If $AB = CD$, then $CD = AB$.	If $m\angle 1 = m\angle 2$, then $m\angle 2 = m\angle 1$.
Transitive	If $AB = CD$ and $CD = EF$, then $AB = EF$.	If $m\angle 1 = m\angle 2$ and $m\angle 2 = m\angle 3$, then $m\angle 1 = m\angle 3$.

Copy and complete

5. Complete the following proof.

Given: $\frac{y+2}{3} = 3$

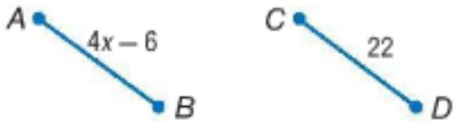
Prove: $y = 7$

Proof:

Statements	Reasons
a. ?	**a.** Given
b. $3\left(\frac{y+2}{3}\right) = 3(3)$	**b.** ?
c. ?	**c.** ?
d. $y = 7$	**d.** Subtraction Property

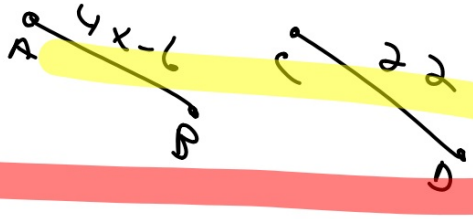
If they give you a conditional statement...
hypothesis: (G)
conclusion: (P)

7. If $\overline{AB} \cong \overline{CD}$, then $x = 7$.



Given: $\overline{AB} \cong \overline{CD}$

Prove: $x = 7$



1. $\overline{AB} \cong \overline{CD}$	1. given
2. $4x - 6 = 22$	2. subs.
3. $4x = 28$	3. add
4. $x = 7$	4. div prop

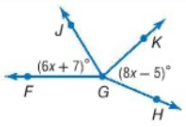
Required: copy ***given, prove, diagram***

where to start? where am I going? map?

Logical, convincing, airtight
Each statement must have a reason

Example 3 Write a Geometric Proof

If $\angle FGJ \cong \angle JGK$ and $\angle JGK \cong \angle KGH$, then $x = 6$.
Write a two-column proof to verify this conjecture.



Given: $\angle FGJ \cong \angle JGK$, $\angle JGK \cong \angle KGH$,
$m\angle FGJ = 6x + 7$, $m\angle KGH = 8x - 5$

Prove: $x = 6$

Proof:

Statements	Reasons
1. $m\angle FGH = 6x + 7$, $m\angle KGH = 8x - 5$ $\angle FGJ \cong \angle JGK$; $\angle JGK \cong \angle KGH$	1. Given
2.	
3.	
4.	
5.	
6.	
7.	
8.	
9.	
10.	
11.	

State the property that justifies each statement.

13. If $5(x + 7) = -3$, then $5x + 35 = -3$.

14. If $m\angle 1 = 25$ and $m\angle 2 = 25$, then $m\angle 1 = m\angle 2$.

15. If $AB = BC$ and $BC = CD$, then $AB = CD$.

16. If $3\left(x - \frac{2}{3}\right) = 4$, then $3x - 2 = 4$.

Copy and

CCSS **ARGUMENTS** Complete each proof.

17. **Given:** $\frac{8-3x}{4} = 32$

Prove: $x = -40$

Proof:

Statements	Reasons
a. $\frac{8-3x}{4} = 32$	**a.** Given
b. $4\left(\frac{8-3x}{4}\right) = 4(32)$	**b.** ?
c. $8 - 3x = 128$	**c.** ?
d. ?	**d.** Subtraction Property
e. $x = -40$	**e.** ?