Geometry 2.3
Analyze statements in if-then form
Write the converse, inverse, and contrapositive of conditional statements

if ... then ...

conditional statement — biconditional "if and only if" — if ⟷ then, both direction (math def)

cause — hypothesis if ( )

happens — conclusion then ( ) — iff

related conditional

converse $H \leftrightarrow C$ — (orig) if Halloween then Oct. T

inverse $\sim H \rightarrow \sim C$ — (converse) if Oct. then Halloween F

contrapositive $\sim C \rightarrow \sim H$ — (inv) if not Hall then not Oct F

logically equivalent — (CP) if not Oct then not Hall T

same T value $H \leftrightarrow C$

OG + CP

KeyConcept Conditional Statement

OG

Words	Symbols
An **if-then statement** is of the form *if p, then q.*	$p \rightarrow q$ read *if p then q*, or *p implies q*
The **hypothesis** of a conditional statement is the phrase immediately following the word *if.*	p
The **conclusion** of a conditional statement is the phrase immediately following the word *then.*	q

2 Related Conditionals

There are other statements that are based on a given conditional statement. These are known as **related conditionals**.

KeyConcept Related Conditionals

Words	Symbols	Examples
T A conditional statement is a statement that can be written in the form *if p, then q.*	OG $p \to q$	If $m\angle A$ is 35, then $\angle A$ is an acute angle.
LE The **converse** is formed by exchanging the hypothesis and conclusion of the conditional.	$q \to p$	If $\angle A$ is an acute angle, then $m\angle A$ is 35. LE
The **inverse** is formed by negating both the hypothesis and conclusion of the conditional.	$\sim p \to \sim q$	If $m\angle A$ is *not* 35, then $\angle A$ is *not* an acute angle.
T The **contrapositive** is formed by negating both the hypothesis and the conclusion of the converse of the conditional.	$\sim q \to \sim p$	If $\angle A$ is *not* an acute angle, then $m\angle A$ is *not* 35.

If you live in Sioux Falls, then you live in SD.

A conditional and its contrapositive are either both true or both false. Similarly, the converse and inverse of a conditional are either both true or both false. Statements with the same truth values are said to be **logically equivalent**.

KeyConcept Logically Equivalent Statements

- A conditional and its contrapositve are logically equivalent.
- The converse and inverse of a conditional are logically equivalent.

cause happens

Example 2 **Write each statement in if-then form.**

5. Sixteen-year-olds are eligible to drive.
6. Cheese contains calcium.
7. The measure of an acute angle is between 0 and 90.
8. Equilateral triangles are equiangular.

5. if 16 then drive
H C

6. if cheese then contains Ca
H C

7. if acute then between 0 + 90
if between 0 + 90 then acute

8. if equil. Then equiang.
if equiang then equil.

if squ has 90°
if 90° then sq.

Example 3 **Determine the truth value of each conditional statement. If *true*, explain your reasoning. If *false*, give a counterexample.**

H C

10. If $x^2 = 16$, then $x = 4$. F CE if $x = -4$

11. If you live in Charlotte, then you live in North Carolina. T

12. If tomorrow is Friday, then today is Thursday. T

13. If an animal is spotted, then it is a Dalmatian. F could be Cheetah

14. If the measure of a right angle is 95, then bees are lizards. T

15. If pigs can fly, then $2 + 5 = 7$. T

Remember: benefit of the doubt...

Whiteboards

C - if ÷4 then ÷2 T
I - if not ÷2 then not ÷4 T
CP if ÷4 then not ÷2 F CE 14

Example 4 CCSS **ARGUMENTS** **Write the converse, inverse, and contrapositive of each true conditional statement. Determine whether each related conditional is *true* or *false*. If a statement is false, find a counterexample.**

16. If a number is divisible by 2, then it is divisible by 4. F CE 6

17. All whole numbers are integers

if whole then int. T

C if int then whole F CE = -6

I if not whole not int. F CE = -12

CP if not int then not whole T

Start by writing in if/then form.

If a triangle is equilateral, then each angle is 60 degrees.
If a triangle has three 60 degree angles, then it is equilateral.

"If $\boxed{\text{and only}}$ if..." iff"

eq iff 3 60°

all 4 rel. cond. T

OG if Δ E then 60° T
C if 60° then Δ E T
I if not Δ E then 60° T
CP if not 60° then not eq Δ T

KeyConcept Biconditional Statement

Words	A biconditional statement is the conjunction of a conditional and its converse.
Symbols	$(p \xrightarrow{T} q) \wedge (q \xrightarrow{T} p) \leftrightarrow (p \leftrightarrow q)$, read *p if and only if q*

If and only if can be abbreviated *iff*.

Examples

Write each biconditional as a conditional and its converse. Then determine whether the biconditional is *true* or *false*. ~~If~~ false, give a counterexample.

a. An angle is a right angle if and only if its measure is 90.

Conditional: If an angle measures 90, then the angle is right.
Converse: If an angle is right, then the angle measures 90.
Both the conditional and the converse are true, so the biconditional is true.

b. $x > -2$ iff x is positive.

Conditional: If x is positive, then $x > -2$. T
Converse: If $x > -2$, then x is positive. F
Let $x = -1$. Then $-1 > -2$, but -1 is not positive. So, the biconditional is false.

Write both statement & converse. Are they both true?

Exercises

Write each biconditional as a conditional and its converse. Then determine whether the biconditional is *true* or *false*. If false, give a counterexample.

1. Two angles are complements if and only if their measures have a sum of 90.
2. There is no school if and only if it is Saturday.

WB 23
pr. 1-7
sk 1-7

if Sat no school T
If no school then Sat F