Geometry 10.7

Find measures of segments that intersect in the interior of a circle

Find measures of segments that intersect in the exterior of a circle

angle = $\frac{1}{2}$(arc + arc)

chord

chord segment

secant

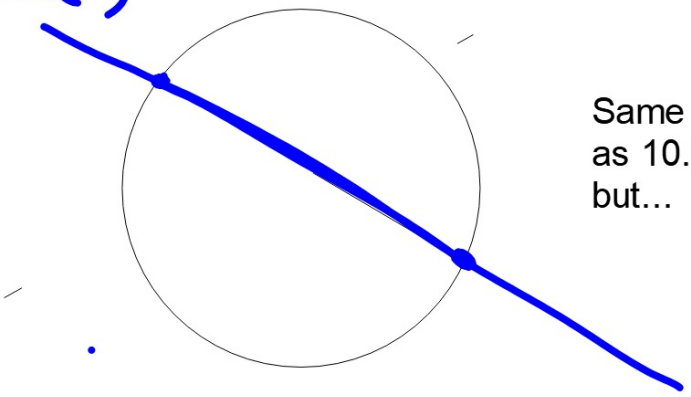
secant segment

(external or internal)

tangent segment

Outside

inside

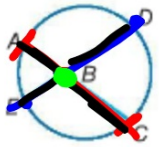


Same picture as 10.6.... but...

Theorem 10.15 Segments of Chords Theorem

Words If two chords intersect in a circle, then the products of the lengths of the chord segments are equal.

Example $AB \cdot BC = DB \cdot BE$



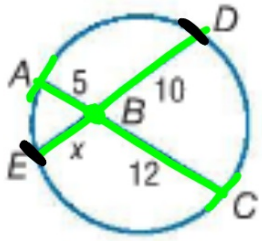
You will prove Theorem 10.15 in Exercise 23.

Based on similar triangles

Example 1 Use the Inter

Find x.

a.



$5 \cdot 12 = 10x$

$60 = 10x$

$6 = x$

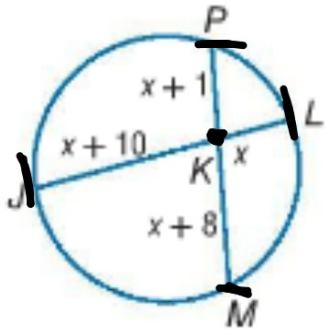
$x + 8$

$x + 1$

$x + 8$

$x^2 + 8x$

b.



$x(x+10) = (x+1)(x+8)$

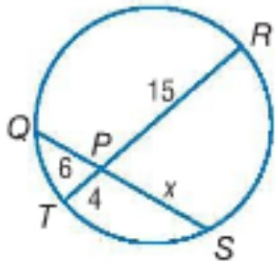
$x^2 + 10x = x^2 + 9x + 8$

$-x^2 - 9x \quad -x^2 - 9x$

$x = 8$

GuidedPractice

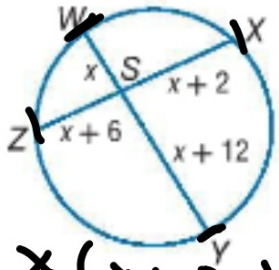
1A.



x^2 $6x$ 12

$x = 3$

1B.



$(x+6)(x+2) = x(x+12)$

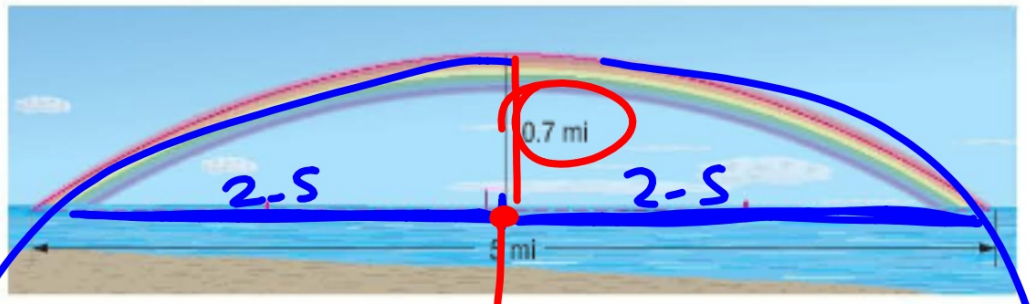
$x^2 + 8x + 12 = x^2 + 12x$

$-x^2 - 8x$

$12 = 4x$

Real-World Example 2 Find Measures of Segments in Circles

SCIENCE The true shape of a rainbow is a complete circle. However, we see only the arc of the circle that appears above Earth's horizon. What is the radius of the circle containing the arc of the rainbow shown?



$2.5(2.5) = 0.7x$

$\frac{6.25}{0.7} = \frac{0.7x}{0.7}$

$8.93 = x$

x

8.93

$d = 9.63$

$r = 4.81$

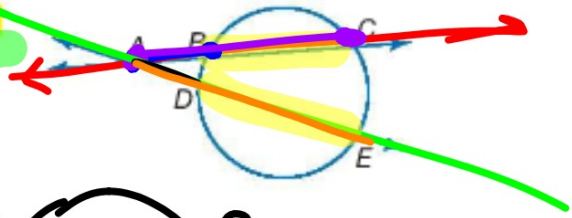
4.8 mi

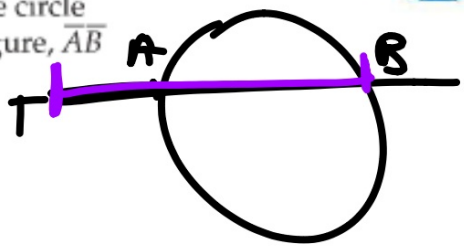
external...internal...whole thing

$\text{angle} = \frac{1}{2}(\text{arc} - \text{arc})$

2 Segments Intersecting Outside a Circle A **secant segment** is a segment of a secant line that has exactly one endpoint on the circle. In the figure, $\overline{AC}$, $\overline{AB}$, $\overline{AE}$ and $\overline{AD}$ are secant segments.

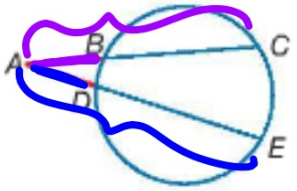
A secant segment that lies in the exterior of the circle is called an **external secant segment**. In the figure, $\overline{AB}$ and $\overline{AD}$ are external secant segments.





Theorem 10.16 Secant Segments Theorem

Words If two secants intersect in the exterior of a circle, then the product of the measures of one secant segment and its external secant segment is equal to the product of the measures of the other secant and its external secant segment.



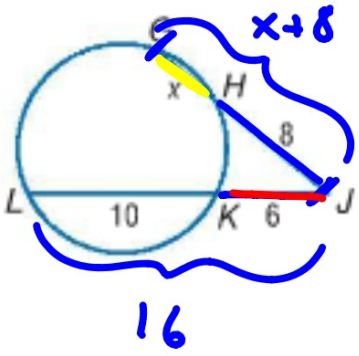
Example $AC \cdot AB = AE \cdot AD$

You will prove Theorem 10.16 in Exercise 24.

external segment x whole thing

$x =$

What is the "whole thing"?



$8(x+8) = 6(16)$

$8x + 64 = 96$

$-64 \quad -64$

$\frac{8x}{8} = \frac{32}{8}$

$x = 4$

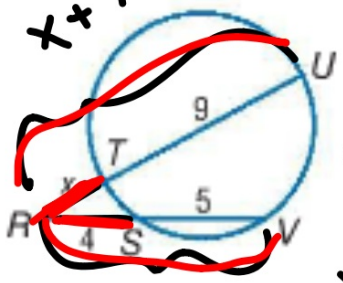
$x(x+9) = 4 \cdot 9$

$x^2 + 9x = 36$

$-36 \quad -36$

GuidedPractice

3A.



9

$x^2 + 9x - 36 = 0$

-36

$12 \quad -3$

9

$(x+12)(x-3) = 0$

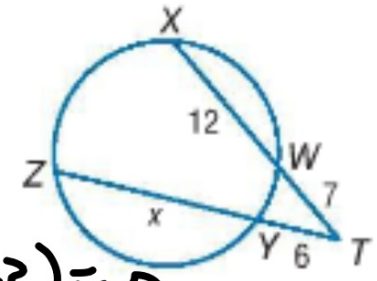
$x+12 = 0$

$x = -12$

$x-3 = 0$

$x = 3$

3B.

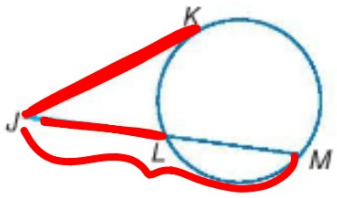


The whole tangent is external...

Theorem 10.17

Words If a tangent and a secant intersect in the exterior of a circle, then the square of the measure of the tangent is equal to the product of the measures of the secant and its external secant segment.

Example $JK^2 = JL \cdot JM$

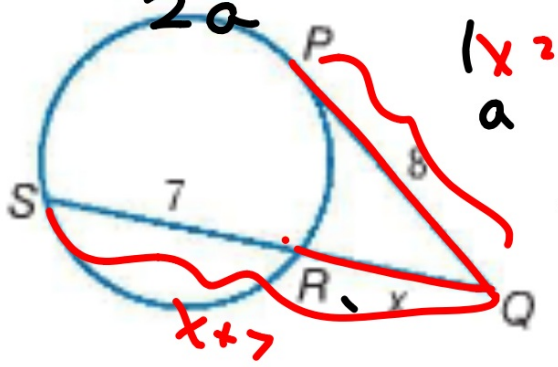


You will prove Theorem 10.17 in Exercise 25.

1 64
2 32
4 16
8 8

ent

$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$



$x = 5.25$

~~$x = +2.25$~~

-64
7

64

$x(x+7) = 8 \cdot 8$

$1x^2+7x-64 = 0$

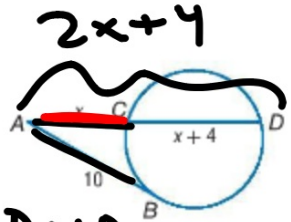
a b c

$x = \frac{-7 \pm \sqrt{49+256}}{2}$

$x = \frac{-7 \pm 17.5}{2}$

GuidedPractice

4. $\overline{AB}$ is tangent to the circle. Find x. Round to the nearest tenth.



$(x)(2x+4) = 10 \cdot 10$

$2x^2 + 4x = 100$

$\frac{2x^2}{2} + \frac{4}{2}x - \frac{100}{2} = \frac{0}{2}$

$x^2 + 2x - 50 = 0$

$x = \frac{-2 \pm \sqrt{4 + 200}}{2}$

$x = \frac{-2 \pm 14.3}{2}$

$\frac{-2 + 14.3}{2}$ $\frac{-2 - 14.3}{2}$

$x = 6.2$

~~$x = -8.2$~~

★ 10.7 p.753
7-21 odd, 22, 37-49 odd