Geometry 5.6

Apply the hinge theorem or its converse to make comparisons in two triangles

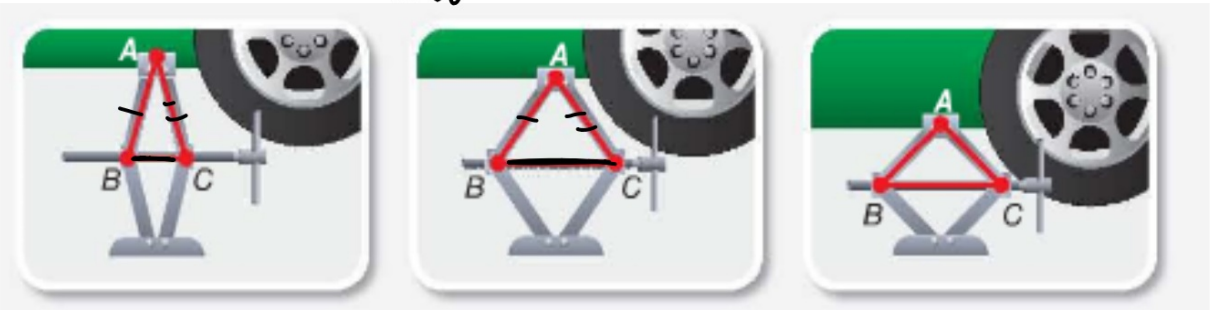
Prove triangle relationships using the hinge theorem for its converse

SSS
SAS
included angle
converse
hinge theorem

activ: coffee stirrers/ spaghetti

Hinge theorem (SAS)

different ∠s



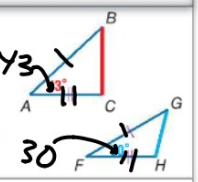
2 same sides...
Included angle changes...
What happens to opposite side?

1 Hinge Theorem The observation in the example above is true of any type of triangle and illustrates the following theorems. Converse

Theorems Inequalities in Two Triangles

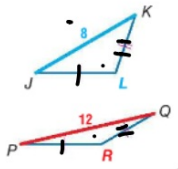
5.13 Hinge Theorem If two sides of a triangle are congruent to two sides of another triangle, and the included angle of the first is larger than the included angle of the second triangle, then the third side of the first triangle is longer than the third side of the second triangle.

Example: If $\overline{AB} \cong \overline{FG}$, $\overline{AC} \cong \overline{FH}$, and $m\angle A > m\angle F$, then $BC > GH$.



5.14 Converse of the Hinge Theorem If two sides of a triangle are congruent to two sides of another triangle, and the third side in the first is longer than the third side in the second triangle, then the included angle measure of the first triangle is greater than the included angle measure in the second triangle.

Example: If $\overline{JL} \cong \overline{PR}$, $\overline{KL} \cong \overline{QR}$, and $PQ > JK$, then $m\angle R > m\angle L$.



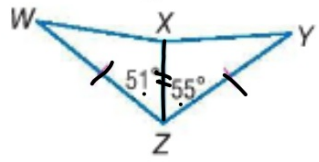
SAS relationship:
The one with the larger included angle...

Not by eyeball...
SAS situation...

Example 1 Use the Hinge Theorem and its Converse

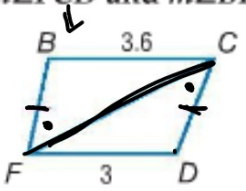
Compare the given measures.

a. WX and XY



$WX < XY$

b. $m\angle FCD$ and $m\angle BFC$

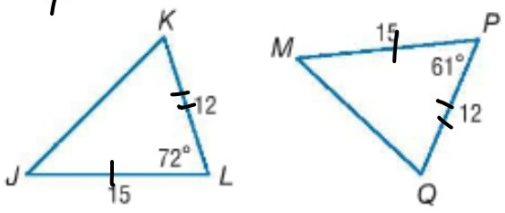


$m\angle FCD < m\angle BF$

GuidedPractice

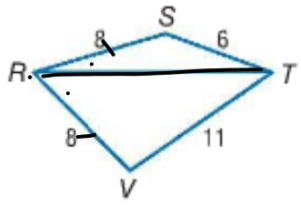
Compare the given measures.

1A. JK and MQ

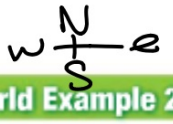


$JK > MQ$

1B. $m\angle SRT$ and $m\angle VRT$

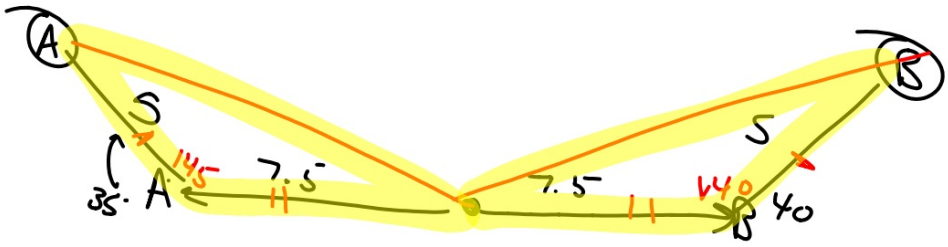


$m\angle SRT < m\angle URT$

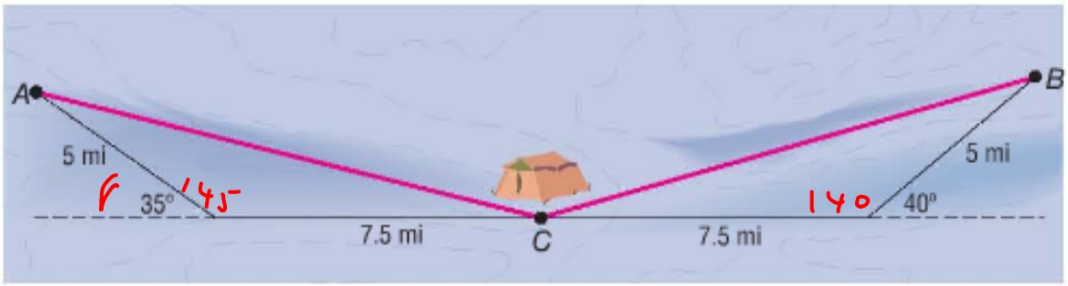


Real-World Example 2 Use the Hinge Theorem

SNOWMOBILING Two groups of snowmobilers leave from the same base camp. Group A goes 7.5 miles due west and then turns 35° north of west and goes 5 miles. Group B goes 7.5 miles due east and then turns 40° north of east and goes 5 miles. At this point, which group is farther from the base camp? Explain your reasoning.



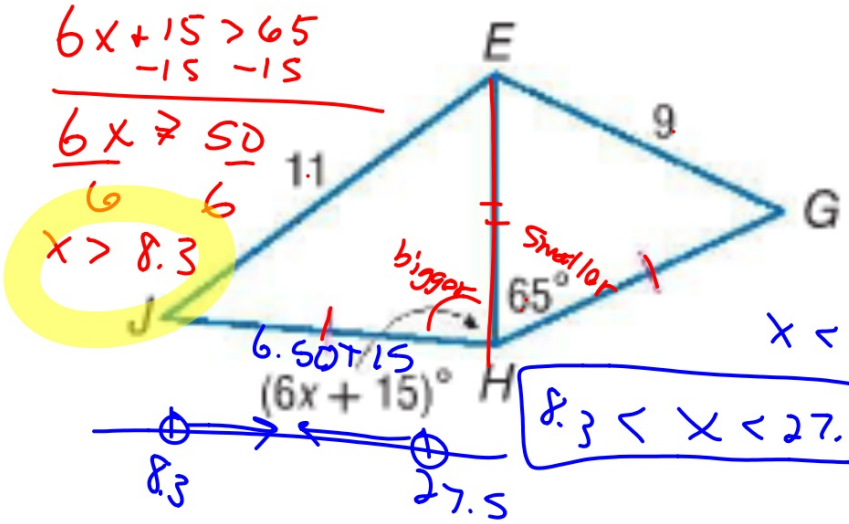
Plan Draw a diagram of the situation.



Solve for $\angle$s $\quad 0 < \text{angle} < 180$

Example 3 Apply Algebra to the Relationships in Triangles

ALGEBRA Find the range of possible values for x.



$6x + 15 > 65$
$-15 \quad -15$
$6x > 50$
$x > 8.3$

biggest

Smallest

$6x + 15 > 0$
$-15 \quad -15$
$\frac{6x}{6} > \frac{-15}{6} \quad x > -3.5$

Which angle is across from the longest side?

$6x + 15 < 180$
$-15 \quad -15$
$\frac{6x}{6} < \frac{165}{6}$

$x < 27.5$

$8.3 < x < 27.5$

StudyTip

Using Additional Facts

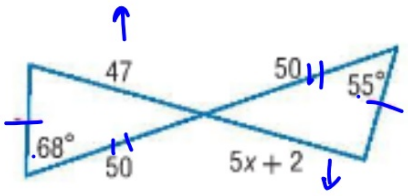
When finding a range for the possible values for x, you may need to use one of the following facts.

- The measure of any angle is always greater than 0 and less than 180.
- The measure of any segment is always greater than 0.

Sides : side > 0

GuidedPractice

3. Find the range of possible values for x.

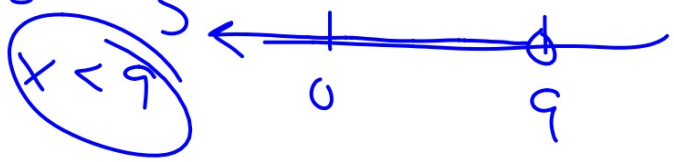


$5x+2 < 47$ $\quad 47 > 5x+2$

$-2 \quad -2$

$\frac{5x}{5} < \frac{45}{5}$ $\quad 0 < x < 9$

$x < 9$



Which side is across from the largest angle?

StudyTip

Using Additional Facts

When finding a range for the possible values for x, you may need to use one of the following facts.

- The measure of any angle is always greater than 0 and less than 180.
- The measure of any segment is always greater than 0.

5.6 p. 376

9-22
31-36
53-55
} all