Geometry 6.3

Recognize conditions that ensure a quadrilateral is a parallelogram

Prove that a set of points forms a parallelogram

converse ⊤

parallelogram properties

coordinate proof

midpoint formula

distance formula

slope formula

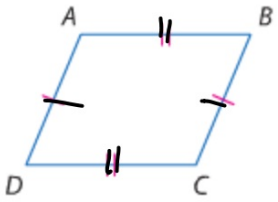
Converse...

Theorems Conditions for Parallelograms

6.9 If both pairs of opposite sides of a quadrilateral are congruent, then the quadrilateral is a parallelogram.

Abbreviation *If both pairs of opp. sides are ≅, then quad. is a ▱.*

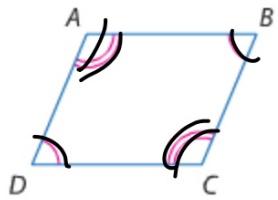
Example If $\overline{AB} \cong \overline{DC}$ and $\overline{AD} \cong \overline{BC}$, then *ABCD* is a parallelogram.



6.10 If both pairs of opposite angles of a quadrilateral are congruent, then the quadrilateral is a parallelogram.

Abbreviation *If both pairs of opp. ∠s are ≅, then quad. is a ▱.*

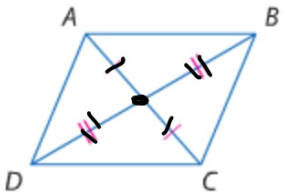
Example If $\angle A \cong \angle C$ and $\angle B \cong \angle D$, then *ABCD* is a parallelogram.



6.11 If the diagonals of a quadrilateral bisect each other, then the quadrilateral is a parallelogram.

Abbreviation *If diag. bisect each other, then quad. is a ▱.*

Example If $\overline{AC}$ and $\overline{DB}$ bisect each other, then *ABCD* is a parallelogram.



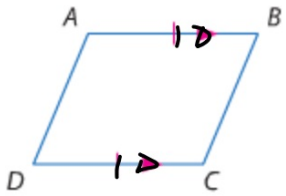
new

SAME pair

6.12 If one pair of opposite sides of a quadrilateral is both parallel and congruent, then the quadrilateral is a parallelogram.

Abbreviation *If one pair of opp. sides is ≅ and ||, then the quad. is a ▱.*

Example If $\overline{AB} \parallel \overline{DC}$ and $\overline{AB} \cong \overline{DC}$, then *ABCD* is a parallelogram.



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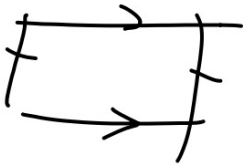
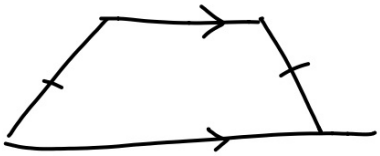


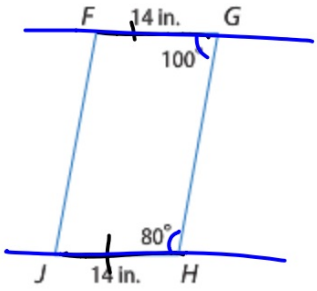


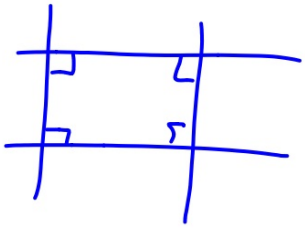
Can't go by eyeball... which theorem/property applies?

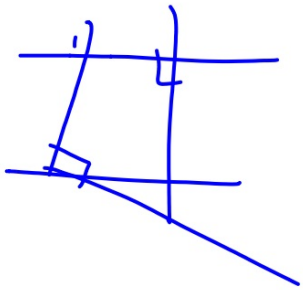
Example 1 Identify Parallelograms

Determine whether the quadrilateral is a parallelogram. Justify your answer.

yes: same pr. sides ≅ & ∥

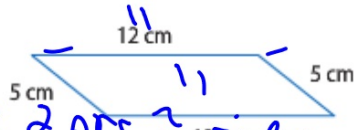






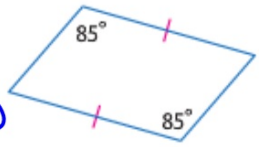
GuidedPractice

1A.



yes: 2 prs ≅ sides

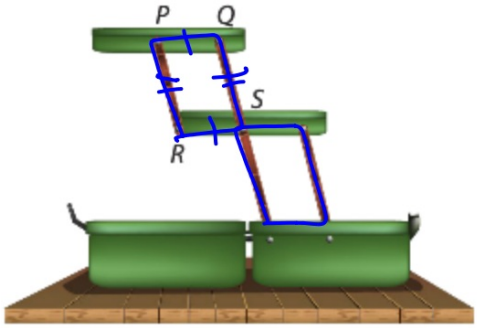
1B.



NO

Real-World Example 2 Use Parallelograms to Prove

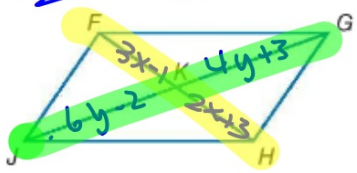
FISHING The diagram shows a side view of the tackle box at the left. In the diagram, $PQ = RS$ and $PR = QS$. Explain why the upper and middle trays remain parallel no matter to what height the trays are raised or lowered.



Real-World Link

A 2- or 3-cantilever tackle box is often used to organize lures and other fishing supplies. The trays lift up and away so that all items in the box are easily accessible.

$6y-2=4y+3$
$-4y+2 \quad -4y+2$
$2y = 5$
$\frac{2y}{2} = \frac{5}{2}$



If I want it to be a parallelogram, what has to be true?

Example 3 Use Parallelograms and Algebra to Find Value

If $FK = 3x - 1$, $KG = 4y + 3$, $JK = 6y - 2$, and $KH = 2x + 3$, find x and y so that the quadrilateral is a parallelogram.

$3x-1=2x+3$
$-2x+1 \quad -2x+1$
$x = 4$

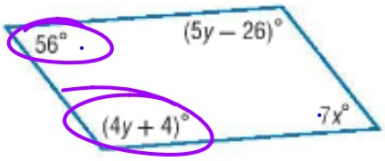
$x = 4$

$y = 2.5$

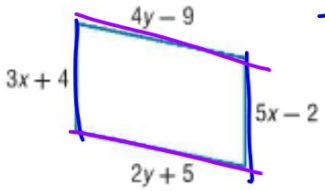
GuidedPractice

Find x and y so that each quadrilateral is a parallelogram.

3A.



3B.



$5x-2=3x+4$
$-3x+2 \quad -3x+2$
$2x=+6$
$x=3$

$56=7x$
$\frac{}{7} \quad \frac{}{7}$
$8=x$

$5y-26=4y+4$
$-4y+26 \quad -4y+26$
$y=30$

$4y-9=2y+5$
$-2y+9 \quad -2y+9$
$2y=14$
$y=7$

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Concept Summary

Prove that a Quadrilateral Is a Parallelogram

- Show that both pairs of opposite sides are parallel.
- Show that both pairs of opposite sides are congruent.
- Show that both pairs of opposite angles are congruent.
- Show that the diagonals bisect each other.
- Show that a pair of opposite sides is both parallel and congruent.

StudyTip

Midpoint Formula

To show that a quadrilateral is a parallelogram, you can also use the Midpoint Formula. If the midpoint of each diagonal is the same point, then the diagonals bisect each other.

Show cons. int supp.

GuidedPractice

Determine whether the quadrilateral is a parallelogram. Justify your answer using the given formula.

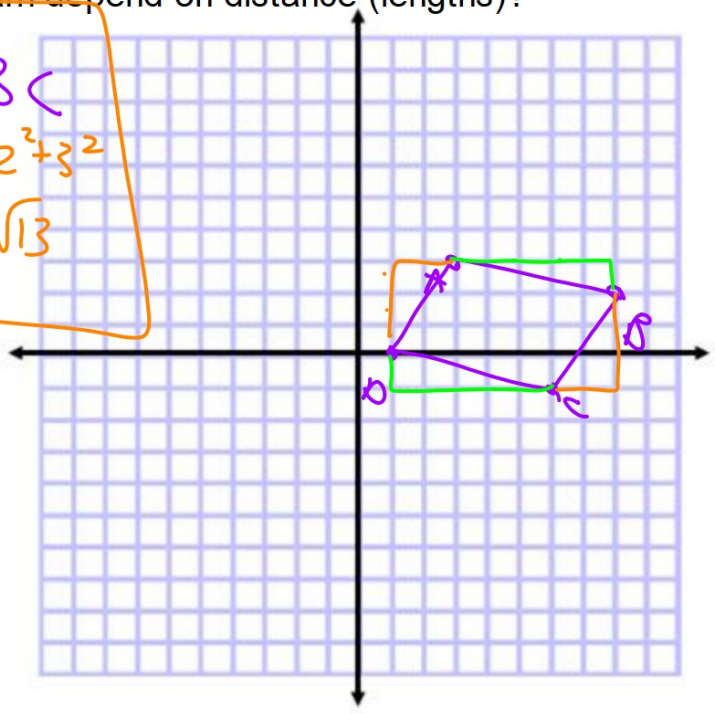
4A. $A(3, 3), B(8, 2), C(6, -1), D(1, 0)$; Dista

What properties of parallogram depend on distance (lengths)?

$AB = CD \quad AD = BC$

$5^2+1^2 \quad 5^2+1^2 \quad 2^2+3^2 \quad 2^2+3^2$

$\sqrt{26} = \sqrt{26} \;||\; \sqrt{13} = \sqrt{13}$



4B $F(-2, 4)$ $G(4, 2)$ $H(4, -2)$ $J(-2, -1)$; Midpoint Formula

What parallelogram properties depend on midpoints?

not a □

mp FH

$\frac{-2+4}{2}$ $\frac{4+-2}{2}$

$\frac{2}{2}$ $\frac{2}{2}$

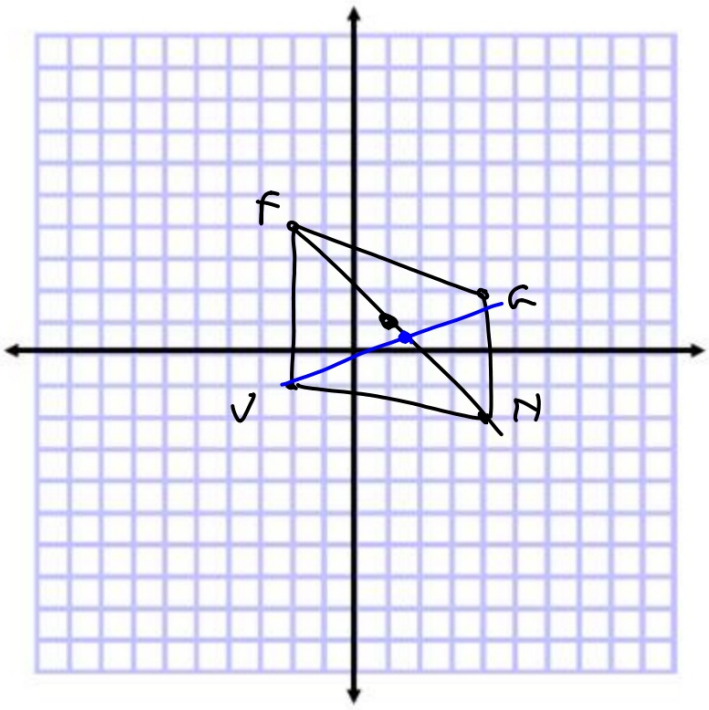
$(1, 1)$

mp GJ

$\frac{4+-2}{2}$ $\frac{2+-1}{2}$

$\frac{2}{2}$ $\frac{1}{2}$

$(1, \frac{1}{2})$



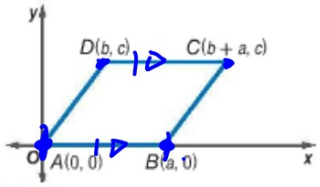
use slopes

Step 2 Use your figure to write a proof.

Given: quadrilateral $ABCD$, $\overline{AB} \parallel \overline{DC}$, $\overline{AB} \cong \overline{DC}$

Prove: $ABCD$ is a parallelogram.

Coordinate Proof:



Hint: Prove that it has 2 pairs of parallel sides

1. $\overline{AB} \parallel \overline{DC}$ $\overline{AB} \cong \overline{DC}$	1. given
2. $m_{AB} = \frac{0}{a} = 0$ $m_{DC} = \frac{c-c}{b+a-b} = 0$	2. def slope
3. $AB = a$ $CD = \frac{b+a-b}{} = a$	3. def length
4. $ABCD$ is a $\square$	4. same pr $\cong$ & $\parallel$

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9-27 odd