

Geometry 12.3

Find lateral areas and surface areas of pyramids.

Find lateral areas and surface areas of cones.

pyramid

apex/vertex *point at top*

regular pyramid

base *any polygon*

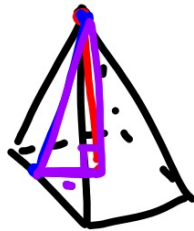
face

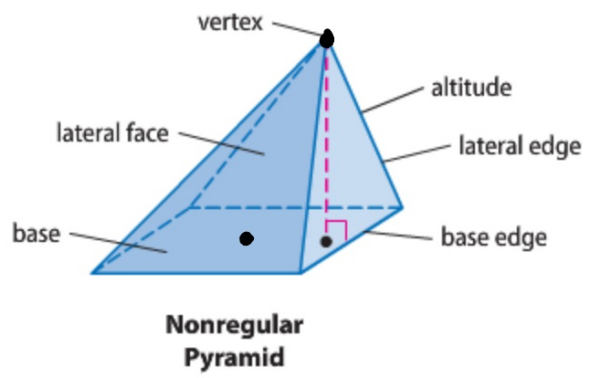
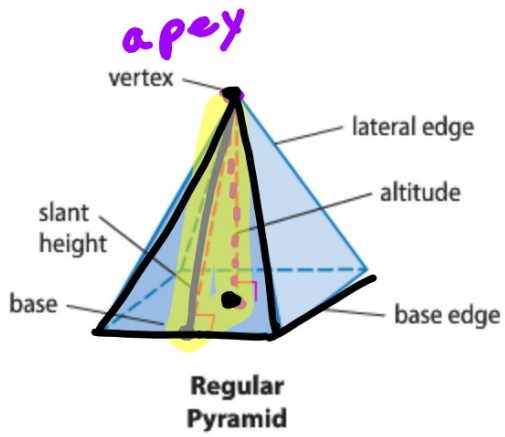
* slant height (l)

* altitude (h)

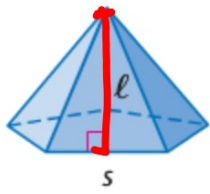
right cone

oblique cone

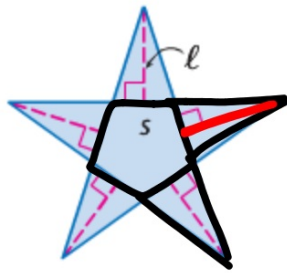




oblique



net



face = isos. \triangle

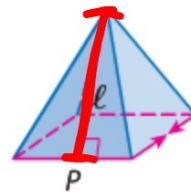
popcorn box



KeyConcept Lateral Area of a Regular Pyramid

Words The lateral area L of a regular pyramid is $L = \frac{1}{2}P\ell$, where ℓ is the slant height and P is the perimeter of the base.

Model



Symbols $L = \frac{1}{2}P\ell$

$$LA: \frac{1}{2} P \ell$$

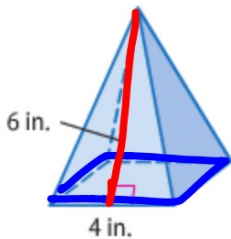
Prisms $LA = pl$sides are rectangles

Pyramids $LA = \frac{1}{2}pl$...sides are triangles

$$SA = LA + B$$

Example 1 Lateral Area of a Regular Pyramid

Find the lateral area of the square pyramid.



$$L = \frac{1}{2} P l$$

$$= \frac{1}{2} \cdot 16 \cdot 6$$

$$= 48 \text{ in}^2$$

$$SA = 48 + 16 =$$

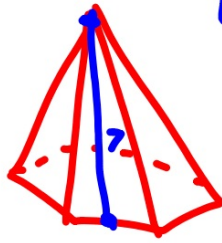
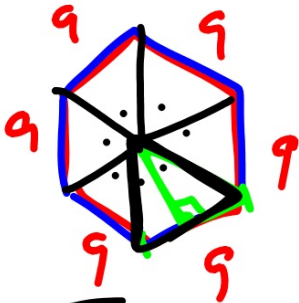
$$\frac{1}{2} P \cdot l$$

$$\frac{1}{2} \cdot 16 \cdot 6$$

$$= 64 \text{ in}^2$$

$$SA = LA + B = 189 + 210.6 = 399.6 \text{ cm}^2$$

1. Find the lateral area of a regular hexagonal pyramid with a base edge of 9 centimeters and a lateral height of 7 centimeters.



$$LA = \frac{1}{2} P \cdot \ell$$

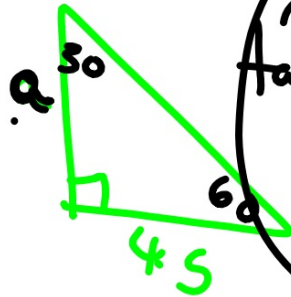
$$= \frac{1}{2} \cdot 54 \cdot 7$$

$$= 189 \text{ cm}^2$$

$$\frac{1}{2} a p$$

$$\frac{1}{2} (7.8)(54)$$

$$210.6$$




$$\tan 60 = \frac{a}{4.5}$$

$$= \frac{a}{4.5}$$

$$a = 4.5(1.73)$$

$$a = 7.8$$

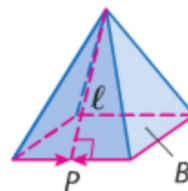
$$\text{Area} = \frac{1}{2} a P \quad \text{apothem}$$

 **Key Concept** Surface Area of a Regular Pyramid

Words

The surface area S of a regular pyramid is $S = \frac{1}{2}P\ell + B$, where P is the perimeter of the base, ℓ is the slant height, and B is the area of the base.

Model



Symbols

$$S = \frac{1}{2}P\ell + B$$

↑ ↑
LA

$$SA = LA + B = 410.16 + 144 = 554.2 \text{ cm}^2$$

Example 2 Surface Area of a Square Pyramid

Find the surface area of the square pyramid to the nearest tenth.

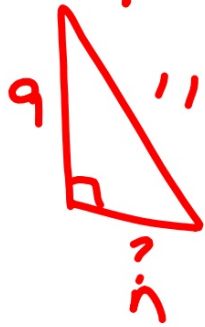
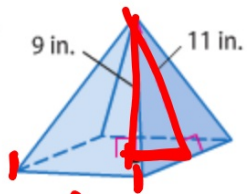


$$\begin{aligned} LA &= \frac{1}{2} p l \\ &= \frac{1}{2} \cdot 48 \cdot 17.09 \\ &= 410.16 \end{aligned}$$

A right-angled triangle drawn in red. The vertical leg is labeled 16, the horizontal leg is labeled 6, and the hypotenuse is labeled 17.09. A right angle symbol is shown at the vertex between the two legs.

Guided Practice

2A.



$$SA = LA + B$$

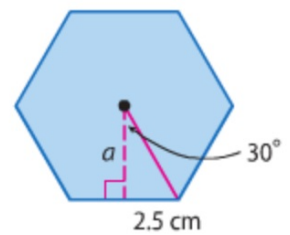
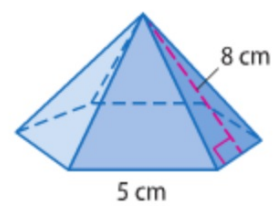
↑ ↑

2B.



Example 3 Surface Area of a Regular Pyramid

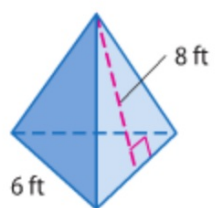
Find the surface area of the regular pyramid. Round to the nearest tenth.



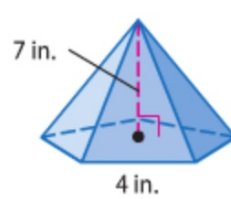
Reg. polygon: $A = (1/2)ap$

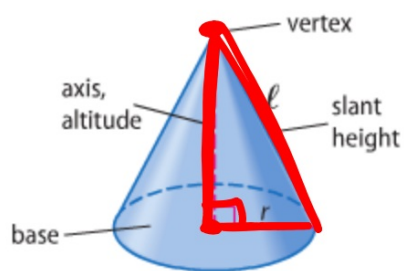
Guided Practice

3A.

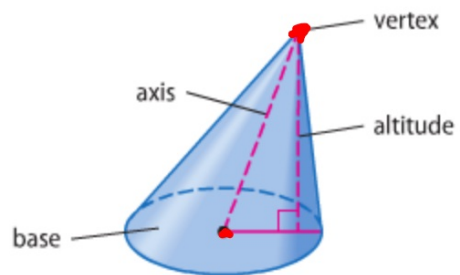


3B.





Right Cone



Oblique Cone



Key Concept Lateral and Surface Area of a Cone

Words The lateral area L of a right circular cone is

$$LA = \frac{1}{2} p \ell$$

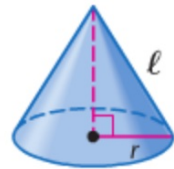
The surface area S of a right circular cone is

$$LA = \frac{1}{2} \cdot \pi d \cdot \ell$$

Symbols

$$SA = LA + B$$

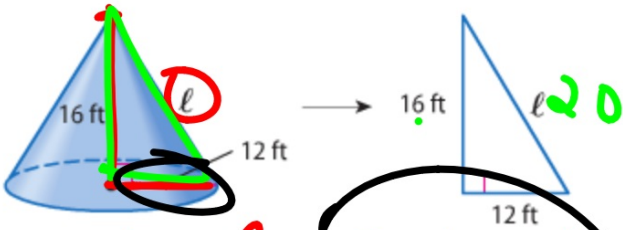
Model



$$SA = LA + B$$

Real-World Example 4 Lateral Area of a Cone

ARCHITECTURE The conical slate roof at the right has a height of 16 feet and a radius of 12 feet. Find the lateral area.



$$LA = \frac{1}{2} p \cdot l$$

$$= \frac{1}{2} (2\pi r) l$$

$$754 \text{ ft}^2$$

$$\pi r^2$$

$$452.39$$

$$1206.4 \text{ ft}^2$$



Guided Practice

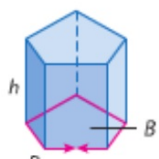
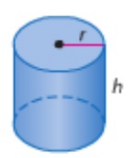
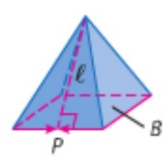
4. **ICE CREAM** A waffle cone is $5\frac{1}{2}$ inches tall and the diameter of the base is $2\frac{1}{2}$ inches. Find the lateral area of the cone. Round to the nearest tenth.

$$\begin{array}{l} P \quad 856 \\ \quad 7-19 \\ \quad \quad 55 \quad 56 \quad 57 \end{array}$$

ConceptSummary Lateral and Surface Areas of Solids

WatchOut!

Bases The bases of right prisms and right pyramids are not always regular polygons.

Solid	Model	Lateral Area	Surface Area
prism		$L = Ph$	$S = L + 2B$ or $S = Ph + 2B$
cylinder			
pyramid		$L = \frac{1}{2}Pl$	$S = \frac{1}{2}Pl + B$
cone	