Algebra 2 3.4

Solve systems of linear equations in 3 variables

Use systems to solve problems

ordered triple (x, y, z) 3 eq.

no solution

infinitely many solutions

substitution

elimination

whiteboards? if time

CK vases

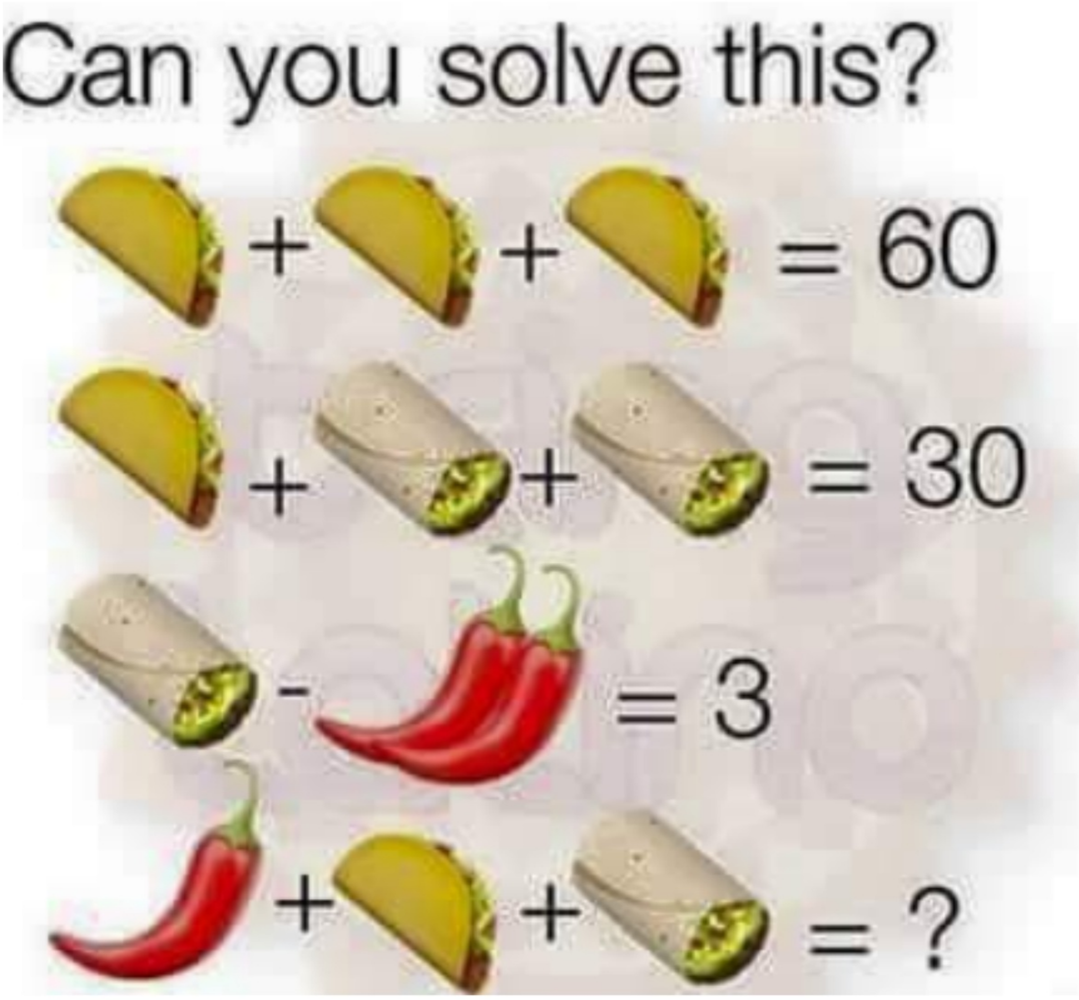
8 simple $\frac{1}{8}$ hr

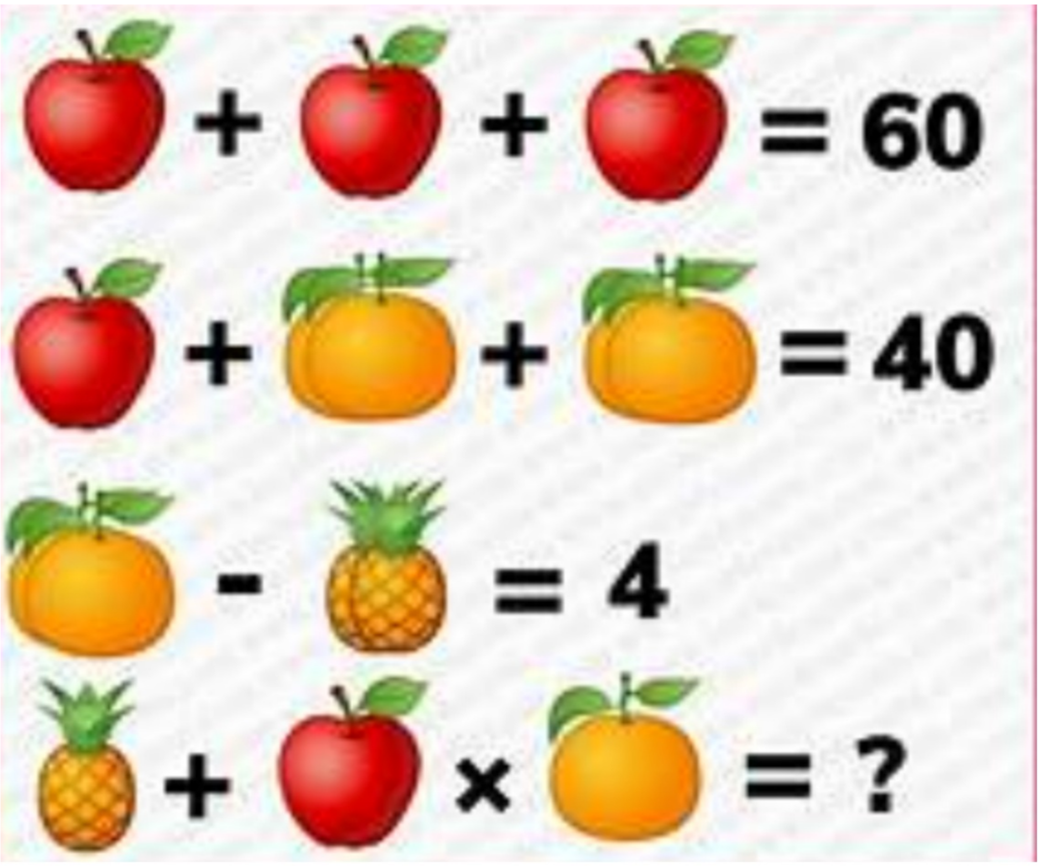
2 elab $\frac{1}{2}$ hr.

$\frac{1}{2}e + \frac{1}{8}s \leq 8$ $e + s \geq 40$

Correct hours

$e + s \leq 8$ $8s + 2e \geq 40$

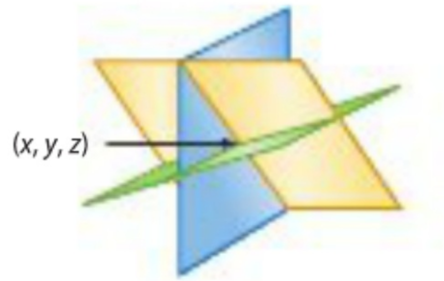

Can you solve this?
+ + = 60
+ + = 30
- = 3
+ + = ?

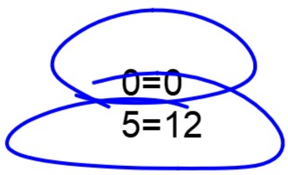

+ + = 60
+ + = 40
- = 4
+ × = ?

One Solution

The three individual planes intersect at a specific point.

(x, y, z)

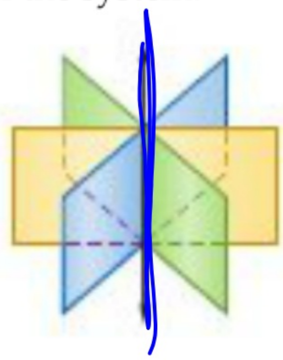



0=0
5=12

Infinitely Many Solutions

The planes intersect in a line.

Every coordinate on the line represents a solution of the system.



The planes intersect in the same plane.

Every equation is equivalent.
Every coordinate in the plane represents a solution of the system.

Don't worry about "which" infinitely many it is...

No Solution There are no points in common with all three planes.

We won't worry about *which* no solution it is.

Be strategic!
1. elimination
2. elim again
(or could use subs)
3. ans. is ordered triple

Example 1 A System with One Solution

Solve the system of equations.

$3 \cdot -1 - 2 \cdot -5 + 4 \cdot 7 = 35$
$-3 + 10 + 28 = 35$

a $3x - 2y + 4z = 35$
b $-4x + y - 5z = -36$ ×2 ×3
c $5x - 3y + 3z = 31$

The coefficient of 1 in the second equation makes y a good choice for elimination.

$3x - 2y + 4z = 35$
$-8x + 2y - 10z = -72$
d $-5x - 6z = -37$ ×-2

$-5 \cdot -1 - 6z = -37$
$5 - 6z = -37$
$-5 \quad -5$
$-6z = -42$

$-12x + 3y - 15z = -108$
$5x - 3y + 3z = 31$
e $-7x - 12z = -77$
$10x + 12z = 74$
$3x = -3$
$x = -1$

(x, y, z)
$(-1, -5, 7)$

$5 \cdot -1 - 3 \cdot y + 3 \cdot 7 = 31$
$-5 - 3y + 21 = 31$
$-3y + 16 = 31$
$-16 \quad -16$
$-3y = 15$
$y = -5$

Strategery: elim is usually a good place to start

Guided Practice

(x, y, z)

1A.

a. $2x + 4y - 5z = 18$ $\times 2$

b. $-3x + 5y + 2z = -27$ $\times 5$

c. $-5x + 3y - z = -17$ $\times 2$

a+b elim z

$$4x + 8y - 10z = 36$$
$$-15x + 25y + 10z = 135$$

d $\quad -11x + 33y = 171$

b+c elim z

$$-3x + 5y + 2z = -27$$
$$-10x + 6y - 2z = -34$$

e $\quad -13x + 11y = -61$

#variables = # equations

- Seats closest to an amphitheater stage cost $30. The seats in the next section cost $25, and lawn seats are $20. There are twice as many seats in section B as in section A. When all 19,200 seats are sold, the amphitheater makes $456,000.

A system of equations in three variables can be used to determine the number of seats in each section.

Whiteboards

1B. $4x - 3y + 6z = 18$
$-x + 5y + 4z = 48$
$6x - 2y + 5z = 0$

Example 2 No Solution and Infinite Solutions

Solve each system of equations.

a. $5x + 4y - 5z = -10$
$-4x - 10y - 8z = -16$
$6x + 15y + 12z = 24$

b. $-6a + 9b - 12c = 21$
$-2a + 3b - 4c = 7$
$10a - 15b + 20c = -30$

Guided Practice

2A. $-4x - 2y - z = 15$
$12x + 6y + 3z = 45$
$2x + 5y + 7z = -29$

2B. $3x + 5y - 2z = 13$

$-5x - 2y - 4z = 20$

$-14x - 17y + 2z = -19$