



Algebra 2 3.5 organizo. data

Analyze data in matrices

Perform algebraic operations on matrices

distributive property

matrix (matrices pl.)

row across

column up + down

element each entry in chart

corresponding elements

dimension $r \times \mathbb{C}$

scale factor

scalar

whiteboards

$$A = \begin{bmatrix} 3 & 0 & 5 \\ 2 & -1 & 3 \end{bmatrix}$$

$$B = \begin{bmatrix} 1 & 0 & 5 \\ 6 & -3 & 4 \end{bmatrix}$$

Square

Sample matrix

$$A = \begin{bmatrix} 3 & 1 \\ 5 & -6 \end{bmatrix}$$

2×2

$$B = \begin{bmatrix} 6 & 0 \\ -5 & -2 \end{bmatrix}$$

2×2

$$C = \begin{bmatrix} 4 & 1 & 2 \\ 3 & 8 & 7 \end{bmatrix}$$

2×3

$$[A] + [B] = \begin{bmatrix} 9 & 1 \\ 0 & -8 \end{bmatrix}$$

$$B + C \quad \wedge$$

2 Algebraic Operations Several algebraic operations can be performed on data that are organized in matrices. Matrices can be added or subtracted if and only if they have the same dimensions.

KeyConcept Adding and Subtracting Matrices

Words To add or subtract two matrices with the same dimensions, add or subtract their corresponding elements.

Symbols

$$\begin{matrix} A & + & B & = & A + B \\ \begin{bmatrix} a & b \\ c & d \end{bmatrix} & + & \begin{bmatrix} e & f \\ g & h \end{bmatrix} & = & \begin{bmatrix} a+e & b+f \\ c+g & d+h \end{bmatrix} \end{matrix}$$

$$\begin{matrix} A & - & B & = & A - B \\ \begin{bmatrix} a & b \\ c & d \end{bmatrix} & - & \begin{bmatrix} e & f \\ g & h \end{bmatrix} & = & \begin{bmatrix} a-e & b-f \\ c-g & d-h \end{bmatrix} \end{matrix}$$

Example

$$\begin{bmatrix} 3 & -5 \\ 1 & 7 \end{bmatrix} + \begin{bmatrix} -2 & -0 \\ -9 & -10 \end{bmatrix} = \begin{bmatrix} 3+2 & -5+0 \\ 1+(-9) & 7+10 \end{bmatrix}$$

(if possible)



Example 2 Add and Subtract Matrices

Find each of the following for $\underline{A} = \begin{bmatrix} 16 & 2 \\ -9 & 8 \end{bmatrix}$, $\underline{B} = \begin{bmatrix} -4 & -1 \\ -3 & -7 \end{bmatrix}$, and $\underline{C} = \begin{bmatrix} 8 \\ 6 \end{bmatrix}$.

a. $A + B$

b. $B - C$

c. $B - A$

$$\begin{bmatrix} -4 & -1 \\ -3 & -7 \end{bmatrix} + \begin{bmatrix} -16 & -2 \\ 9 & -8 \end{bmatrix} =$$

SAME dimensions!

whiteboards

Guided Practice

2A. $\begin{bmatrix} -3 & 4 \\ -9 & -5 \end{bmatrix} - \begin{bmatrix} -4 & 12 \\ 8 & -7 \end{bmatrix}$

2C. $\begin{bmatrix} 8 & -3 \\ -2 & 0 \\ 1 & 7 \end{bmatrix} - \begin{bmatrix} 5 & 1 & -4 & 2 \\ 10 & -6 & 9 & 0 \end{bmatrix}$

3×2 (2×4)

$$3+3+3+3+3 = 15 \quad \text{Scalar mult.}$$

$$5 \cdot 3 =$$

$$A = \begin{bmatrix} 1 & 2 \\ 5 & 0 \end{bmatrix} \quad 2 \times 2$$

$$3 \cdot A = \begin{bmatrix} 3 & 6 \\ 15 & 0 \end{bmatrix} \quad 2 \times 2$$

Example 3 Multiply a Matrix by a Scalar

If $R = \begin{bmatrix} -12 & 8 & 6 \\ -16 & 4 & 19 \end{bmatrix}$, find $5R$.

Scalar...scale factor

Guided Practice

3. If $T = \begin{bmatrix} 8 & 0 & 3 & -2 \\ -1 & -4 & -2 & 9 \end{bmatrix}$, find $-4T$.

KeyConcept Properties of Matrix Operations

For any matrices A , B , and C for which the matrix sum and product are defined and any scalar k , the following properties are true.

Commutative Property of Addition

$$A + B = B + A$$

✓

Associative Property of Addition

$$(A + B) + C = A + (B + C)$$

✓

Left Scalar Distributive Property

$$k(A + B) = kA + kB$$

✓

Right Scalar Distributive Property

$$(A + B)k = kA + kB$$

✓

scalar multiplication is not the same as matrix multiplication...

scalar $\cdot$ matrix

$$k \begin{bmatrix} \\ \end{bmatrix}$$

matrix $\cdot$ matrix

$$\begin{bmatrix} \\ \end{bmatrix} \cdot \begin{bmatrix} \\ \end{bmatrix}$$

dimensions

Order of operations.... GEMA

Example 4 Multi-Step Operations

If $A = \begin{bmatrix} -9 & 12 \\ 2 & -6 \end{bmatrix}$ and $B = \begin{bmatrix} -4 & -8 \\ 2 & -3 \end{bmatrix}$, find $-4B + 3A$.

2×2 2×2

Guided Practice

4. If $A = \begin{bmatrix} -5 & 3 \\ 6 & -8 \\ 2 & 9 \end{bmatrix}$ and $B = \begin{bmatrix} 12 & 5 \\ 5 & -4 \\ 4 & -7 \end{bmatrix}$, find $-6B + 7A$.

3×2 3×2

$$\begin{matrix} A & B \\ \begin{bmatrix} 1 & 3 \\ 2 & 6 \end{bmatrix} & \begin{bmatrix} 1 & 4 & 7 \\ 3 & 1 & 2 \end{bmatrix} \end{matrix}$$

$$2 \times 2 \quad 2 \times 3 \quad \text{yes} \quad \text{ans. } 2 \times 3$$

$$\begin{bmatrix} 1 & 4 \\ 3 & 7 \\ 0 & 5 \end{bmatrix} \cdot \begin{bmatrix} 3 & 1 \\ 2 & 0 \end{bmatrix} \quad \text{yes} \quad \text{ans } 3 \times 2$$

$$3 \times 2 \quad 2 \times 2$$

$$\begin{bmatrix} 1 & 0 & 5 \end{bmatrix} \cdot \begin{bmatrix} 2 & 0 & 6 \end{bmatrix} \quad \text{no}$$

$$1 \times 3 \quad 1 \times 3$$

$$1 \times 3 \quad 3 \times 1$$

$$\begin{bmatrix} 0 & 5 \\ 1 & 3 \end{bmatrix} \cdot \begin{bmatrix} 3 & 0 \\ 1 & 5 \end{bmatrix} = \text{yes} \quad 2 \times 2$$

$$2 \times 2 \quad 2 \times 2$$

$$\begin{bmatrix} \quad \end{bmatrix} \cdot \begin{bmatrix} \quad \end{bmatrix} \quad \text{yes} \quad 1 \times 2$$

$$1 \times 3 \quad 3 \times 2$$