Algebra 2 3.8
Find the inverse of a 2x2 matrix
Write and solve matrix equations

square matrix
identity matrix
inverse matrix
matrix equation
coefficient matrix
variable matrix
constant matrix
whiteboards

Wife Swap (TV?)

Matrix equation is different than Cramer's rule...
(Most students have a favorite after they try both methods.)

$[A]^{-1}$

2 × 2 Identity Matrix

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

3 × 3 Identity Matrix

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

KeyConcept Identity Matrix for Multiplication

Words The identity matrix for multiplication I is a square matrix with 1 for every element of the main diagonal, from upper left to lower right, and 0 in all other positions. For any square matrix A of the same dimension as I, $A \cdot I = I \cdot A = A$.

Symbols If $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, then $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ such that

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}.$$

$5 \cdot 2 \cdot \frac{1}{2} =$

Two $n \times n$ matrices are **inverses** of each other if their product is the identity matrix. If matrix A has an inverse symbolized by A^{-1}, then $A \cdot A^{-1} = A^{-1} \cdot A = I$.

$-1+1$

Is their product the identity matrix?

Example 1 Verify Inverse Matrices

Determine whether the matrices in each pair are inverses.

a. $A = \begin{bmatrix} -4 & 2 \\ -2 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} \frac{1}{4} & -\frac{1}{2} \\ \frac{1}{2} & -1 \end{bmatrix}$

2×2 $\quad$ 2×2

$A \cdot B = \begin{bmatrix} 0 & \\ & \end{bmatrix}$

no

$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

yes

$2+2$

Determine whether the matrices in each pair are inverses.

1. $A = \begin{bmatrix} 2 & 1 \\ -1 & 0 \end{bmatrix}, B = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$

2. $C = \begin{bmatrix} 2 & 1 \\ 5 & 3 \end{bmatrix}, D = \begin{bmatrix} 2 & 1 \\ 5 & -3 \end{bmatrix}$

$AB =$

$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

no

KeyConcept **Inverse of a 2 × 2 Matrix**

The inverse of matrix $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is $A^{-1} = \frac{1}{ad - bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$, where $ad - bc \neq 0$.

$$\frac{1}{det}\begin{bmatrix} \text{Wife} \\ \text{Swap} \end{bmatrix}$$

What is ad-bc?
1/det

$$\frac{1}{det}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix} =$$

Wife Swap: TV show

Example 2 Find the Inverse of a Matrix

Find the inverse of each matrix, if it exists.

a. $P = \begin{bmatrix} 7 & -5 \\ 2 & -1 \end{bmatrix}$ $\quad -7$ $\quad --5$ $\quad -10$ $\quad +10$ $\quad \frac{1}{det}[\text{WS}]$ $\quad P^{-1} = \begin{bmatrix} -\frac{1}{3} & \frac{5}{3} \\ -\frac{2}{3} & \frac{7}{3} \end{bmatrix}$

$\frac{1}{3}\begin{bmatrix} -1 & 5 \\ -2 & 7 \end{bmatrix}$

Why would an inverse not exist?

−92

−72 + +72

b. $Q = \begin{bmatrix} -8 & -6 \\ 12 & 9 \end{bmatrix} = \frac{1}{0}$

−72

NP

whiteboards

Guided Practice

2A. $\begin{bmatrix} 3 & 7 \\ 1 & -4 \end{bmatrix}$

2B. $\begin{bmatrix} 2 & 1 \\ -4 & 3 \end{bmatrix}$

Find the inverse of each matrix, if it exists.

5. $\begin{bmatrix} 6 & -3 \\ -1 & 0 \end{bmatrix}$

6. $\begin{bmatrix} 2 & -4 \\ -3 & 0 \end{bmatrix}$

Different than Cramer's rule...

Start

2 Matrix Equations

Matrices can be used to represent and solve systems of equations. You can write a **matrix equation** to solve the system of equations below.

$$\begin{aligned} x + 2y &= 9 \\ 3x - 6y &= 3 \end{aligned} \quad \rightarrow \quad \begin{bmatrix} x + 2y \\ 3x - 6y \end{bmatrix} = \begin{bmatrix} 9 \\ 3 \end{bmatrix}$$

3, 8
13 - 25 0
43 - 55 0

coefficient matrix
variable matrix
constant matrix

To solve: $A^{-1} \times B$

Use a matrix equation to solve each system of equations.

9. $-2x + y = 9$
$x + y = 3$

10. $4x - 2y = 22$
$6x + 9y = -3$