Algebra 2 3.7

Evaluate determinants

Use Cramer's rule to solve systems of equations

Quiz 3.5-3.6 today

matrix

~~square matrix~~

determinant

second order determinant

third order determinant

diagonal method, cofactor method, echelon method

Cramer's rule

coefficient matrix

$\begin{bmatrix} 3 & 5 \\ 1 & 0 \end{bmatrix}$

3 x 3

whiteboards

KeyConcept Second-Order Determinant

Words	The value of a second-order determinant is the difference of the products of the two diagonals.
Symbols	$\det \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$
Example	$\begin{vmatrix} 4 & 5 \\ -3 & 6 \end{vmatrix} = 4(6) - (5)(-3) = 39$

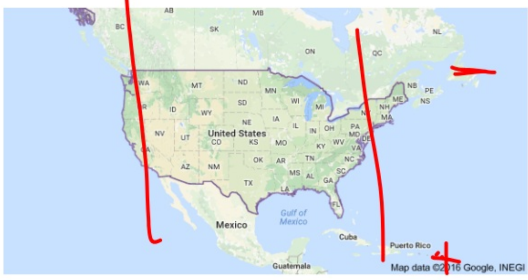
-15

24

$24 - {}^{-}15 = 39$

Notice how the notation has changed...
Does it look like something else?
context...

$[\quad] \qquad |\quad| = \det.$



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Example 1 Second-Order Determinant

Evaluate each determinant.

a. $\begin{vmatrix} 5 & -4 \\ 8 & 9 \end{vmatrix}$ -32

45

$45 - {}^{-}32 = 77$

b. $\begin{vmatrix} 0 & 6 \\ 4 & -11 \end{vmatrix}$ 24

$= -24$

0

GuidedPractice

1A. $\begin{vmatrix} -6 & -7 \\ 10 & 8 \end{vmatrix}$ -70

-48

-48+70

=22

1B. $\begin{vmatrix} 7 & 5 \\ 9 & -4 \end{vmatrix}$

There are multiple methods to evaluate a determinant for 3x3
(We are going to learn the diagonal method...for now.)

Example 2 Use Diagonals

Evaluate $\begin{vmatrix} 4 & -8 & 3 \\ -3 & 2 & 6 \\ -4 & 5 & 9 \end{vmatrix}$ using diagonals.

$$\begin{matrix} 4 & -8 & 3 & 4 & -8 \\ -3 & 2 & 6 & -3 & 2 \\ -4 & 5 & 9 & -4 & 5 \end{matrix}$$

$(-24+120+216)=312$

(312)

$219-312=-93$

(219)

$(72+192+^{-}45)=219$

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Guided Practice

Evaluate each determinant.

2A. $\begin{vmatrix} -5 & 9 & 4 \\ -2 & -1 & 5 \\ -4 & 6 & 2 \end{vmatrix}$ **2B.** $\begin{vmatrix} -8 & -4 & 4 \\ 0 & -5 & -8 \\ 3 & 4 & 1 \end{vmatrix}$

$16 + {}^{-}150 + {}^{-}36 = -170$

$$\begin{array}{ccccc} -5 & 9 & 4 & -5 & 9 \\ -2 & -1 & 5 & -2 & -1 \\ -4 & 6 & 2 & -4 & 6 \end{array}$$

${}^{-}218 + {}^{+}170 = -48$

$10 + {}^{-}180\ {}^{-}48 = -218$

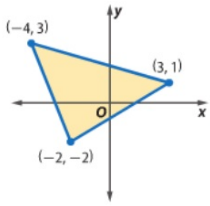
KeyConcept Area of a Triangle

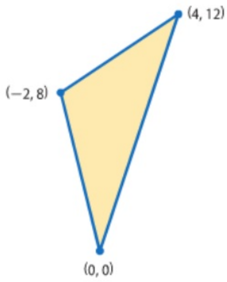
Words The area of a triangle with vertices (a, b), (c, d), and (e, f) is $|A|$, where

$$A = \frac{1}{2}\begin{vmatrix} a & b & 1 \\ c & d & 1 \\ e & f & 1 \end{vmatrix}.$$

Example

$$A = \frac{1}{2}\begin{vmatrix} -4 & 3 & 1 \\ 3 & 1 & 1 \\ -2 & -2 & 1 \end{vmatrix}$$





KeyConcept Cramer's Rule

Let C be the coefficient matrix of the system $\begin{aligned} ax + by &= m \\ fx + gy &= n \end{aligned} \rightarrow \begin{bmatrix} a & b \\ f & g \end{bmatrix}$.

The solution of this system is $x = \dfrac{\begin{vmatrix} m & b \\ n & g \end{vmatrix}}{|C|}$ and $y = \dfrac{\begin{vmatrix} a & m \\ f & n \end{vmatrix}}{|C|}$, if $|C| \neq 0$.

coefficient matrix
variable matrix (either x or y)

Example 4 Solve a System of Two Equations

Solve the system by using Cramer's Rule.

$5x - 6y = 15$
$3x + 4y = -29$

$\begin{bmatrix} 5 & 15 \\ 3 & -29 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 15 \\ -29 \end{bmatrix}$

Test-Taking Tip

Cramer's Rule When the determinant of the coefficient matrix C is 0, the system does not have a unique solution.

$x \quad y$

$\left(\frac{-114}{38}, \frac{-190}{38} \right)$

20 ~~+~~ -18

60 - 174

-145 - 45

$(-3, -5)$

$\begin{bmatrix} -3 \\ -5 \end{bmatrix}$

Zero in the denominator...

$$5x + 6y = 20$$

$$-3x - 7y = -29$$

$$\begin{bmatrix} 5 & 20 \\ -3 & -29 \end{bmatrix} \quad \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 20 \\ -29 \end{bmatrix} \qquad \left(\frac{34}{-17}, \frac{-85}{-17} \right)$$

$$-35 + 18 \qquad -145 + 60$$

$$-140 + 174$$

$$\begin{bmatrix} -2 \\ 5 \end{bmatrix}$$