Algebra 2 4.4

Perform operations with imaginary numbers*

Perform operations with complex numbers*

radical

simplify (by casting out pairs) geometry

square root property

real number

imaginary unit

pure imaginary numbers

complex numbers

complex conjugate

$\sqrt{-1} = i$

$-1 = (i \cdot i)$

$-1i = i \cdot i \cdot i$

$1 = (i\,i\,i\,i)$

*New concept--first time you have seen this idea!

No solution = no real solution!

-432

$16 \quad 27$

$4 \quad 4 \quad 3 \quad 9$

$2 \quad 2 \quad 2 \quad 2 \quad 3 \quad 3$

$$\sqrt{-18} \cdot \sqrt{-24}$$

$$\sqrt{18} \cdot i\sqrt{24}\, i$$

$$(2 \cdot 2)3\sqrt{3}\, i i$$

$$-12\sqrt{3}$$

4

$\frac{9}{4} = 2 R 1$

i^{103} $25 R 3$

$-1 i = -i$ $(i\, i) i = -i$

$i^{9} = (i\, i)(i\, i)(i\, i)(i\, i)\, i = i$ $i^{96} = 1$

$1 \cdot 1\ i$

$\frac{31}{4} = 7 R 3$ $24 R 0$

i^{31} $(i\, i) i = -i$

$(1 \cdot 1 \cdot 1 \cdot 1 \cdot 1 \cdot 1 \cdot 1)$

i^{96}

$y = x^2 + 64$

Example 3 Equation with Pure Imaginary Solutions

Solve $x^2 + 64 = 0$.

$-64 \quad -64$

$\sqrt{x^2} = \sqrt{-64}$

$x = \pm 8i$

64
8 8

64
16 4
4 4 2 2
2 2 2 2

$2 \cdot 2 \cdot 2$

GuidedPractice

Solve each equation.

3A. $4x^2 + 100 = 0$

$$4x^2 = -100$$
$$\frac{4x^2}{4} = \frac{-100}{4}$$
$$\sqrt{x^2} = \sqrt{-25}$$
$$x = \pm 5i$$

3B. $x^2 + 4 = 0$

$5 + 3i$

2 Operations with Complex Numbers

Consider $2 + 3\boldsymbol{i}$. Since 2 is a real number and $3\boldsymbol{i}$ is a pure imaginary number, the terms are not like terms and cannot be combined. This type of expression is called a **complex number**.

KeyConcept Complex Numbers

Words	A complex number is any number that can be written in the form $a + b\boldsymbol{i}$, where a and b are real numbers and $\boldsymbol{i}$ is the imaginary unit. a is called the real part, and b is called the imaginary part.
Examples	$5 + 2\boldsymbol{i}$ $1 - 3\boldsymbol{i} = 1 + (-3)\boldsymbol{i}$

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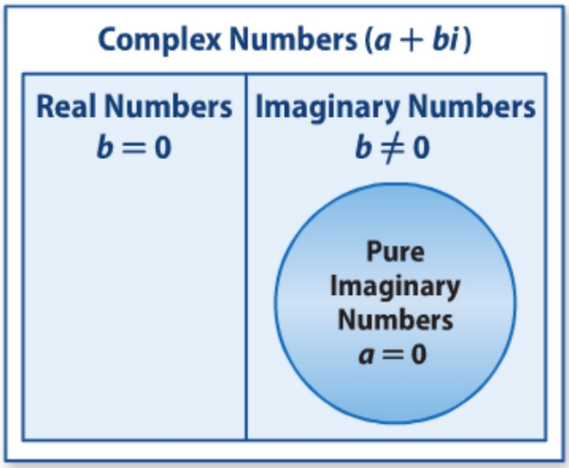
The Venn diagram shows the set of complex numbers.

- If $b = 0$, the complex number is a real number.
- If $b \neq 0$, the complex number is imaginary.
- If $a = 0$, the complex number is a pure imaginary number.



Two complex numbers are equal if and only if their real parts are equal and their imaginary parts are equal. That is, $a + bi = c + di$ if and only if $a = c$ and $b = d$.

real = real   imag = imag $(4, 9)$

Example 4 Equate Complex Numbers

Find the values of x and y that make $3x - 5 + (y - 3)i = 7 + 6i$ true.

$$3x - 5 = 7$$
$$+5 \quad +5$$
$$3x = 12$$
$$x = 4$$

$$(y - 3)i = 6i$$
$$yi - \cancel{3i} = 6i$$
$$+\cancel{3i} \quad +3i$$
$$\frac{y \cdot i}{i} = \frac{9i}{i}$$
$$y = 9$$

StudyTip

Reading Math Electrical engineers use j as the imaginary unit to avoid confusion with the i for current.

$2+3i \quad 2+3j$

Complex numbers are used with electricity. In these problems, j usually represents the imaginary unit. In a circuit with alternating current, the voltage, current, and impedance, or hindrance to current, can be represented by complex numbers. To multiply these numbers, use the FOIL method.

Real-World Example 6 Multiply Complex Numbers

ELECTRICITY In an AC circuit, the voltage V, current C, and impedance I are related by the formula $V = C \cdot I$. Find the voltage in a circuit with current $2 + 4j$ amps and impedance $9 - 3j$ ohms.

$V = C \cdot I$

$? = (2+4j)(9-3j)$

$V = 30 + 30j$

$$\begin{array}{r} 2+4i \\ 9-3i \\ \hline -6i \quad -12ii \\ 18 \quad 36i \\ \hline 18+12 \end{array}$$

$$(11-8i) \overset{-}{+}{}^{-1}(2-8i)$$

$$11-8i-2+8i$$

$$9+0i$$

$$9$$

$3+i$
$3-i$
$-3i \quad -ii$
$9 \quad 3i$
$9+1$

$$\frac{5}{3+i}\cdot\frac{(3-i)}{3-i} \qquad \frac{15-5i}{10}(-1) \qquad \frac{3-i}{2}(-1) \qquad \boxed{\frac{3}{2}-\frac{i}{2}}$$

$$\frac{2}{3+\sqrt{5}}\cdot\frac{(3-\sqrt{5})}{3-\sqrt{5}} = \frac{6-2\sqrt{5}}{9-5} = \frac{6-2\sqrt{5}}{4} = \boxed{\frac{3-\sqrt{5}}{2}}$$

$$\frac{3}{2-5i} \cdot \frac{(2+5i)}{2+5i} = \frac{6+15i}{29}$$

$$\frac{6}{29} + \frac{15i}{29}$$

$$\frac{2+5i}{7-3i}\cdot\frac{7+3i}{7+3i} = \frac{-1+41i}{58} = \frac{-1}{58} + \frac{41i}{58}$$

WB odds
+32

$7-3i$
$7+3i$

$+21i \quad -9ii$
$49 \quad -21i \quad -9(-1)$

$2+5i$
$7+3i$

$6i + 15ii$
$14 \quad 35i$

$14 + 41i + -15$