Algebra 2 4.4

Perform operations with imaginary numbers*

Perform operations with complex numbers*

radical

simplify (by casting out pairs) geometry

square root property

real number

imaginary unit

pure imaginary numbers

complex numbers

complex conjugate

$\sqrt{49} = 7$

$\sqrt{-4} = (\ )(\ )$

no real

*New concept--first time you have seen this idea!

No solution = no real solution!

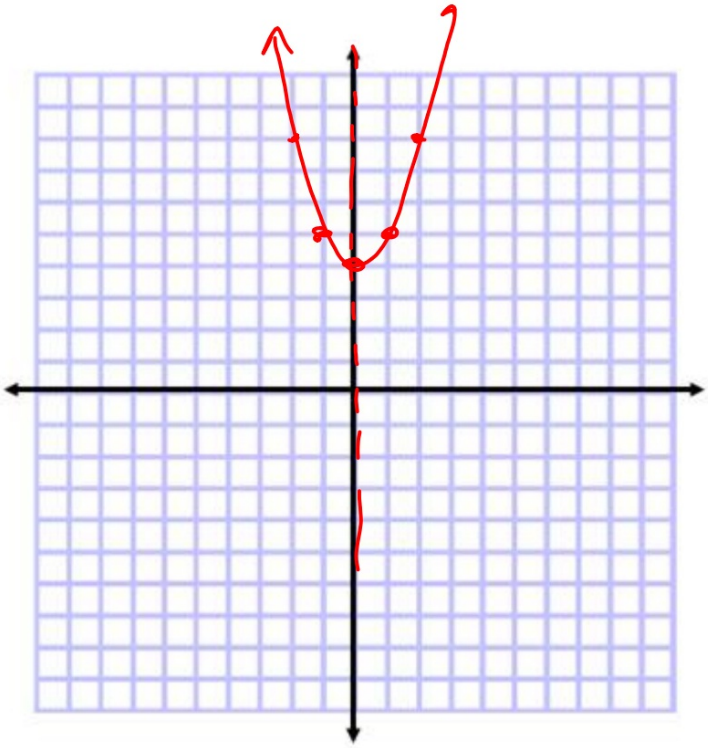
Solve by graphir

$x^2 + 4 = 0$

$y = x^2 + 4$

$x = -\frac{0}{2\cdot 1} = 0$

0	0^2+4	4
1	$1+4$	5
2	$4+4$	8



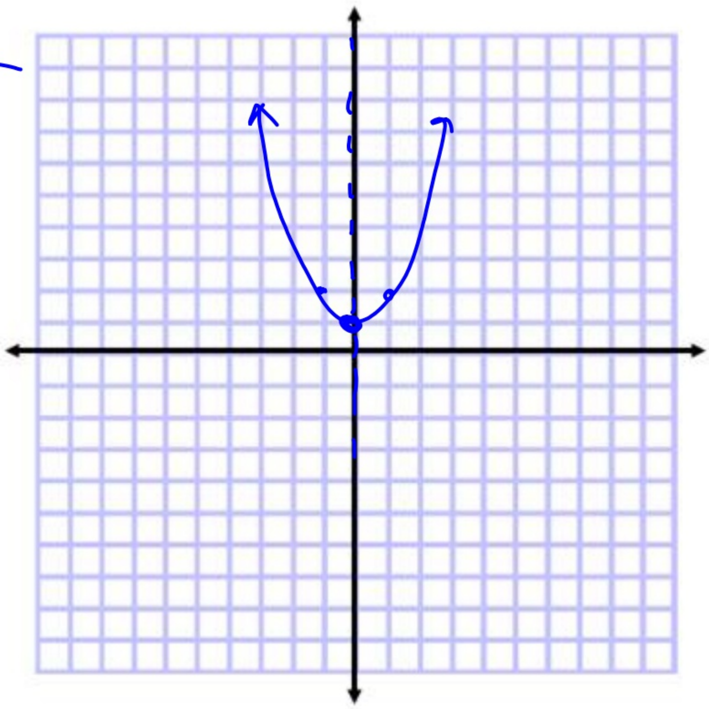
$x^2 + 1 = 0$

Graph the related function
Solve using algebra

$y = x^2 + 1$

$x = \frac{-0}{2}$

0	0+1	1
1	1+1	2



The **imaginary unit i** is defined to be

$$\boxed{i = \sqrt{-1}.}$$

$\sqrt{-1}$ = imaginary
= i

$(\;)\cdot(\;) = -1$

Numbers of the form $6i$, $-2i$, and $i\sqrt{3}$ are called **pure imaginary numbers**. Pure imaginary numbers are square roots of negative real numbers. For any positive real number b, $\sqrt{-b^2} = \sqrt{b^2} \cdot \sqrt{-1}$ or bi.

$6i$

$2 + 3i$
complex number

Pure	Complex
$2i$	$2+3i$
$5i$	$5-2i$
$-6i$	$7+\sqrt{5}i$
$\sqrt{7}i$	

Guided Practice Simplify

1A. $\sqrt{-18}$

$\sqrt{18} \cdot \sqrt{-1}$

18
2 9
1 3 3

$3\sqrt{2} \cdot \sqrt{-1}$

$3\sqrt{2} \cdot i = 3i\sqrt{2}$

1B. $\sqrt{-125}$

125
5 25
5 5

$\sqrt{125} \cdot \sqrt{-1}$

$5\sqrt{5} \cdot \sqrt{-1}$

$5\sqrt{5}i = 5i\sqrt{5}$

"Casting out pairs" to simplify the real part
What about the negative?

$i^1 = \sqrt{-1}$

$i^2 = i \cdot i = \sqrt{-1} \cdot \sqrt{-1} = -1$

$i^3 = i \cdot i \cdot i = \underbrace{(\sqrt{-1} \cdot \sqrt{-1})}_{-1} \underbrace{\sqrt{-1}}_{i} = -1i$

$i^4 = iiii = \underbrace{\sqrt{-1} \cdot \sqrt{-1}}_{-1} \cdot \underbrace{\sqrt{-1} \cdot \sqrt{-1}}_{-1} = 1$

$\sqrt{-250}$

$\boxed{\sqrt{250} \cdot \sqrt{-1}}$

$5\sqrt{10}\, i =$

$\cancel{5\sqrt{10}i}$

$$\boxed{\begin{aligned} i &= \sqrt{-1} \\ i^2 &= -1 \\ i^3 &= -i \\ i^4 &= 1 \end{aligned}}$$

128

$2 \wedge 64$

$8 \wedge 8$

$4 \wedge 2 \quad 4 \wedge 2$

$2 \wedge 2 \quad 2\,2$

$\sqrt{-128}$

$\sqrt{128}\ \sqrt{-1}$

$2 \cdot 2 \cdot 2\sqrt{2}\, i$

$8\sqrt{2}\, i =$

$8i\sqrt{2}$

Example 2 Products of Pure Imaginary Numbers

Simplify.

a. $-5i \cdot 3i$ $= -15 \cdot \underbrace{i \cdot i}$

$= -15 \cdot -1$

$= 15$

GuidedPractice

2A. $3i \cdot 4i$

$12ii$

$12 \cdot -1$

-12

240

24 10

6 4 2 5

2 3 2 2

$\sqrt{4} = 2$

$\sqrt{2 \cdot 2} = 2$

2B. $\sqrt{-20} \cdot \sqrt{-12}$

$\sqrt{20}\, i \cdot \sqrt{12}\, i$

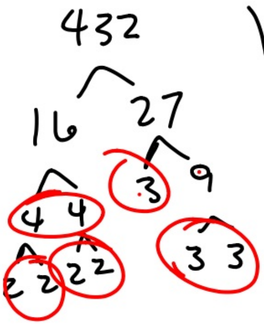
$\sqrt{240}\, ii$

$(2 \cdot 2)\sqrt{15}\,(ii)$

$-4\sqrt{15}$

2C. i^{31}

$4 \cdot 4 \left(\begin{matrix} 1-5 \\ 18-24 \end{matrix} \right)$



$\sqrt{-18} \cdot \sqrt{-24}$

$\sqrt{18} \cdot i \sqrt{24}\, i$

$(2 \cdot 2) 3 \sqrt{3}\, i i$

$-12\sqrt{3}$

4

Example 3 Equation with Pure Imaginary Solutions

Solve $x^2 + 64 = 0$.

GuidedPractice

Solve each equation.

3A. $4x^2 + 100 = 0$

3B. $x^2 + 4 = 0$

2 Operations with Complex Numbers Consider $2 + 3i$. Since 2 is a real number and $3i$ is a pure imaginary number, the terms are not like terms and cannot be combined. This type of expression is called a **complex number**.

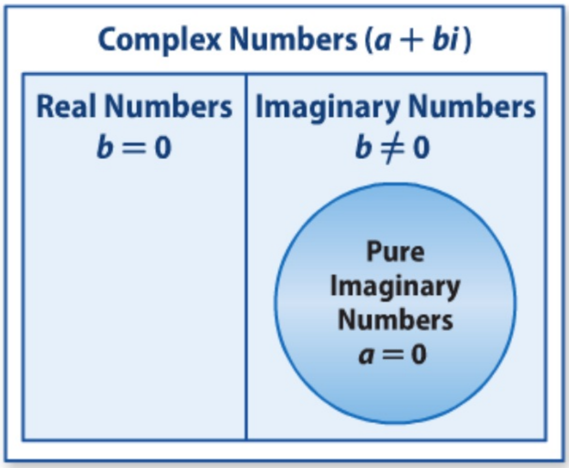
KeyConcept Complex Numbers

Words	A complex number is any number that can be written in the form $a + bi$, where a and b are real numbers and i is the imaginary unit. a is called the real part, and b is called the imaginary part.
Examples	$5 + 2i$ $1 - 3i = 1 + (-3)i$

The Venn diagram shows the set of complex numbers.

- If $b = 0$, the complex number is a real number.
- If $b \neq 0$, the complex number is imaginary.
- If $a = 0$, the complex number is a pure imaginary number.

Two complex numbers are equal if and only if their real parts are equal and their imaginary parts are equal. That is, $a + b\boldsymbol{i} = c + d\boldsymbol{i}$ if and only if $a = c$ and $b = d$.



real = real   imag = imag

Example 4 Equate Complex Numbers

Find the values of x and y that make $3x - 5 + (y - 3)i = 7 + 6i$ true.