Algebra 2 4.6

Solve quadratic equations by using the quadratic formula

Use the discriminant to determine the number and type of roots for a quadratic equation

standard form (of a quadratic) $ax^2+bx+c=0$

discriminant b^2-4ac pos → 2 real (ps.) rational; neg → 2 imag (cat)

quadratic formula $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

complex number

conjugate pair

irrational number

exact answer

QF song

whiteboards

KeyConcept Quadratic Formula

Words The solutions of a quadratic equation of the form $ax^2 + bx + c = 0$, where $a \neq 0$, are given by the following formula.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Example $x^2 + 5x + 6 = 0 \rightarrow x = \dfrac{-5 \pm \sqrt{5^2 - 4(1)(6)}}{2(1)}$

QF song!

GuidedPractice

Solve each equation by using the Quadratic Formula.

4A. $3x^2 + 5x + 4 = 0$

4B. $x^2 - 4x = -13$

$x^2 - 4x + 13 = 0$

$a = 1 \quad b = -4 \quad c = 13$

$$x = \frac{4 \pm \sqrt{-4 \cdot -4 - 4 \cdot 1 \cdot 13}}{2 \cdot 1} = \frac{4 \pm \sqrt{16 - 52}}{2}$$

$$x = \frac{4 \pm \sqrt{-36}}{2} = \frac{4 \pm 6i}{2} = 2 \pm 3i$$

2 Roots and the Discriminant In the previous examples, observe the relationship between the value of the expression under the radical and the roots of the quadratic equation. The expression $b^2 - 4ac$ is called the **discriminant**.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad \leftarrow \text{discriminant}$$

$d = b^2 - 4ac$

Just the part in RED!

$\frac{3 \pm i\sqrt{87}}{6} = \frac{3}{6} + \frac{i\sqrt{87}}{6}$ $\quad 87 \quad 3$

$= \frac{1}{2} \pm \frac{i\sqrt{87}}{6}$

Example 5 Describe Roots

Find the value of the discriminant for each quadratic equation. Then describe the number and type of roots for the equation.

a. $7x^2 - 11x + 5 = 0$

a b c

$b^2 - 4ac$

$-11 \cdot -11 - 4 \cdot 7 \cdot 5$

$121 - 140$

$d = -20$ 2 imag/comp.

$\frac{-11 \pm \sqrt{-20}}{14}$

b. $x^2 + 22x + 121 = 0$

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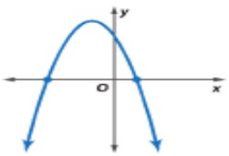
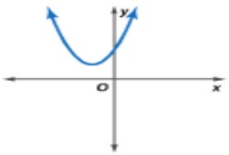
KeyConcept Discriminant

Consider $ax^2 + bx + c = 0$, where a, b, and c are rational numbers and $a \neq 0$.

Value of Discriminant	Type and Number of Roots	Example of Graph of Related Function
$b^2 - 4ac > 0$; $b^2 - 4ac$ is a perfect square. pos.	2 real, rational roots	
$b^2 - 4ac > 0$; $b^2 - 4ac$ is *not* a perfect square. pos	2 real, irrational roots	
$b^2 - 4ac = 0$	1 real rational root	 double root
$b^2 - 4ac < 0$ neg	2 complex roots	

vertex

$$x = \frac{-b}{2a} \pm \left(\frac{\sqrt{b^2 - 4ac}}{2a} \right)$$

Vertex $\pm$ ( )

x-int.

GuidedPractice

5A. $-5x^2 + 8x - 1 = 0$

5B. $-7x + 15x^2 - 4 = 0$

ConceptSummary Solving Quadratic Equations

Method	Can be Used	When to Use
graphing	sometimes	Use only if an exact answer is not required. Best used to check the reasonableness of solutions found algebraically.
factoring	sometimes	Use if the constant term is 0 or if the factors are easily determined. **Example** $x^2 - 7x = 0$
Square Root Property SRP	sometimes	Use for equations in which a perfect square is equal to a constant. **Example** $(x - 5)^2 = 18$
completing the square CTS	always	Useful for equations of the form $x^2 + bx + c = 0$, where b is even. **Example** $x^2 + 6x - 14 = 0$
Quadratic Formula QF	always	Useful when other methods fail or are too tedious. **Example** $2.3x^2 - 1.8x + 9.7 = 0$

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WB pr. odds +30

5.6