Algebra 2 4.6

Solve quadratic equations by using the quadratic formula

Use the discriminant to determine the number and type of roots for a quadratic equation

standard form (of a quadratic)
a,b,c
radical
discriminant
quadratic formula
complex number
conjugate pair
irrational number
exact answer
QF song

$$\boxed{ax^2+bx+c=0}$$

1. graphing
2. CTS
3. factor
4. QF

$\sqrt{-25} = 5i$

120

12 10

4 3 2 5

2 2

$\sqrt{-120}$

$\sqrt{4}$

$2i\sqrt{30}$

$\sqrt{80} = 4\sqrt{5}$

16 5

$3+\sqrt{5}$ $3-\sqrt{5}$

$5-2i$ $5+2i$

$$\boxed{ax^2+bx+c=0}$$

$$\frac{2x^2}{2} + \frac{8x}{2} - \frac{6}{2} = \frac{0}{2}$$

$$x^2+4x-3=0$$
$$+3 \quad +3$$

$$x^2+4x+4 = 3+4$$

$$\sqrt{(x+2)^2} = \sqrt{7}$$

$$x+2 = \pm\sqrt{7}$$
$$-2 \quad -2$$
$$x = -2 \pm \sqrt{7}$$

Solve by completing the square
(keep track of the steps used)

1. $\div$ a
2. move c
3. $\frac{1}{2} \cdot b$
4. $(\ )^2$ to both
5. $\sqrt{\ }$ $\pm$
6. x =

Same steps

1 Quadratic Formula You have found solutions of some quadratic equations by graphing, by factoring, and by using the Square Root Property. There is also a formula that can be used to solve any quadratic equation. This formula can be derived by solving the standard form of a quadratic equation.

General Case

$$\frac{ax^2}{a} + \frac{bx}{a} + \frac{c}{a} = \frac{0}{a}$$

$$x^2 + \frac{b}{a}x + \frac{c}{a} = 0$$

$$-\frac{c}{a} \qquad -\frac{c}{a}$$

$$\frac{b}{a} \cdot \frac{1}{2}$$

$$\frac{b}{2a} \cdot \frac{b}{2a}$$

$$x^2 + \frac{b}{a}x + \frac{b^2}{4a^2} = -\frac{c}{a}\cdot\frac{4a}{4a} + \frac{b^2}{4a^2}$$

$$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a^2}$$

$$x + \frac{b}{2a} = \sqrt{\frac{b^2 - 4ac}{4a^2}}$$

$$x + \frac{b}{2a} = \pm\frac{\sqrt{b^2 - 4ac}}{2a}$$

$$-\frac{b}{2a} \qquad -\frac{b}{2a}$$

$$x = \frac{-b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}$$

$$\boxed{x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}}$$

$\cancel{(-2,-3)}$ $\{x \mid x=-2 \quad x=-3\}$

KeyConcept Quadratic Formula

Words The solutions of a quadratic equation of the form $ax^2 + bx + c = 0$, where $a \neq 0$, are given by the following formula. $25-24$

$$\boxed{x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}}$$

Example $x^2 + 5x + 6 = 0 \rightarrow x = \dfrac{-5 \pm \sqrt{5^2 - 4(1)(6)}}{2(1)} = \dfrac{-5 \pm \sqrt{1}}{2}$

$a=1 \quad b=5 \quad c=6$

$$\frac{-5 \pm 1}{2} = \frac{-4}{2} \quad \frac{-6}{2}$$

$x=-2, \; x=-3$

Quadratic formula song

QF song!

Must be in standard form... =0

Example 1 Two Rational Roots

Solve $x^2 - 10x = 11$ by using the Quadratic Formula.

$-11 \quad -11$

$$x^2 - 10x - 11 = 0$$

$$x = \frac{10 \pm \sqrt{-10 \cdot -10 - 4 \cdot 1 \cdot -11}}{2 \cdot 1}$$

$$x = \frac{10 \pm \sqrt{100 + 44}}{2}$$

$$x = \frac{10 \pm 12}{2}$$

$$\frac{10 + 12}{2} \qquad x = 11$$

$$\frac{10 - 12}{2} \qquad x = -1$$

GuidedPractice

Solve each equation by using the Quadratic Formula.

1A. $x^2 + 6x = 16$

1B. $2x^2 + 25x + 33 = 0$

$(x+4)(x+4)=0$

$x+4=0$

Example 2 One Rational Root

$x=-4$ $x=-4$

Solve $x^2 + 8x + 16 = 0$ by using the Quadratic Formula.

$a=1$ $b=8$ $c=16$

$$x = \frac{-8 \pm \sqrt{64-4\cdot 1\cdot 16}}{2}$$

$$= \frac{-8 \pm \sqrt{0}}{2} = \frac{-8 \pm 0}{2}$$

$$-\frac{8}{2} = -4$$

Solve each equation by using the Quadratic Formula.

2A. $x^2 - 16x + 64 = 0$

2B. $x^2 + 34x + 289 = 0$

Example 3 Irrational Roots

Solve $2x^2 + 6x - 7 = 0$ by using the ~~Quadratic Formula~~.

a b c

$$= \frac{-6 \pm \sqrt{36 - 4 \cdot 2 \cdot -7}}{4}$$

$$= \frac{-6 \pm \sqrt{36 + 56}}{4}$$

$$= \frac{-6 \pm \sqrt{92}}{4}$$

$$\frac{-6 \pm 2i\sqrt{23}}{4}$$

$$-\frac{6}{4} \pm \frac{2i\sqrt{23}}{4}$$

$$-\frac{3}{2} \pm \frac{i\sqrt{23}}{2} = \cancel{\frac{-3 \pm \sqrt{23}}{2}}$$

92

46 2

2 23

Guided Practice

Solve each equation by using the Quadratic Formula.

3A. $3x^2 + 5x + 1 = 0$

3B. $x^2 - 8x + 9 = 0$

Example 4 Complex Roots

Solve $x^2 - 6x = -10$ by using the Quadratic Formula.

GuidedPractice

Solve each equation by using the Quadratic Formula.

4A. $3x^2 + 5x + 4 = 0$

4B. $x^2 - 4x = -13$

2 Roots and the Discriminant

In the previous examples, observe the relationship between the value of the expression under the radical and the roots of the quadratic equation. The expression $b^2 - 4ac$ is called the **discriminant**.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \leftarrow \textbf{discriminant}$$

(What is UNDER the radical?
It's a COMPONENT of the QF))

$b^2 - 4ac = 49$

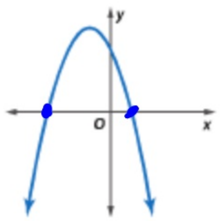
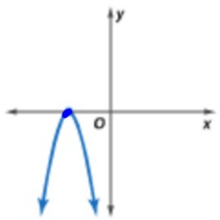
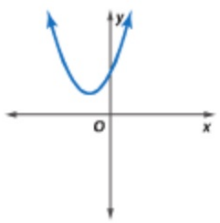
neg = 2 complex (imag)

pos = 2 real → PS = rat.
not PS = irrat

0 = 1 real (D.R.)
rational

KeyConcept Discriminant

Consider $ax^2 + bx + c = 0$, where a, b, and c are rational numbers and $a \neq 0$.

Value of Discriminant	Type and Number of Roots	Example of Graph of Related Function
$b^2 - 4ac > 0$; $b^2 - 4ac$ is a perfect square.	2 real, rational roots	
$b^2 - 4ac > 0$; $b^2 - 4ac$ is *not* a perfect square.	2 real, irrational roots	
$b^2 - 4ac = 0$	1 real rational root	
$b^2 - 4ac < 0$	2 complex roots	

√

(double root)

$x^2 + 3x - 14 = 0$

$9 - 4 \cdot 1 \cdot -14$

$9 + 56$

a) $d = 65$

b) 2 real irrat

c) $x = \frac{-3 \pm \sqrt{65}}{2}$

p 270

15-330

35-40 all