Algebra 2 $(\ )^x$ 7.4

Solve logarithmic equations

Solve logarithmic inequalities

$= \quad < \quad \leq \quad > \quad \geq$

$\log_3(9) = 2$

domain

base $x>0$ $\quad AL>0$

— extraneous (solution)

argument

Richter Scale

$x^3 x^5 = x^8$ $\quad (x^4)^3 =$

$\frac{x^7}{x^3} = x^4$

Charles Richter, c. 1970

Officially:

$$M_{\mathrm{L}} = \log_{10} A - \log_{10} A_0(\delta) = \log_{10}[A/A_0(\delta)],$$

A, Ao, = distance from seismograph, etc.

Example 1 Solve a Logarithmic Equation

Solve $\log_{36} x = \frac{3}{2}$.

$36^{\frac{3}{2}} = x$

$36^{1.5} = x$

$x = 216$

$\log_{10} 100 = 2$

$10^2 = 100$

Exponential form
domain x>0 Why???
always verify solutions

Solve each equation.

1A. $\log_9 x = \frac{3}{2}$

$9^{\frac{3}{2}} = x$

$x = 27$

1B. $\log_{16}(x) = \frac{5}{2}$

$16^{2.5} = 1024$

KeyConcept Property of Equality for Logarithmic Functions

Symbols	If b is a positive number other than 1, then $\log_b x = \log_b y$ if and only if $x = y$.
Example	If $\log_5 x = \log_5 8$, then $x = 8$. If $x = 8$, then $\log_5 x = \log_5 8$.

$x = 8$

$16^{2.5}$

$16 \cdot 16 \cdot 4$

Are they equal?
Are the bases the same?

Then numbers (antilogs) are the same.

(*Remember: Log is code for exponent*)

Are bases equal?
Are the numbers equal?
Then the exponents are equal!

Standardized Test Example 2 Solve a Logarithmic Equation

Solve $\log_2 (x^2 - 4) = \log_2 3x$.

A -2 B -1 C 2 D 4

$x^2 - 4 = 3x$

$x^2 - 3x - 4 = 0$

$(x-4)(x+1) = 0$

$x = 4$ $x = -1$

Might be extraneous solutions: argument must be positive (Why?)

GuidedPractice

2. Solve $\log_3 (x^2 - 15) = \log_3 2x$.

F -3 **G** -1 **H** 5 **J** 15

$x^2 - 15 = 2x$

$x^2 - 2x - 15 = 0$

$(x-5)(x+3) = 0$

$x = 5$ $x = -3$

Example 3 Solve a Logarithmic Inequality

Solve $\log_3 x < 4$.

$x < 3^4$

$x < 81$

$x > 0$

My number...their number

Argument must be positive.

Is my number (x) bigger or smaller than 3^4?

How can I tell?

GuidedPractice

Solve each inequality.

3A. $\log_4 x \geq 3$

$x \geq 4^3$

$x \geq 64$

My number...their number

Is my number bigger or smaller than 4^3?

3B. $\log_2(x) < 4$

$x < 2^4 \quad 0 < x < 16$

$x < 16$

My number...their number

Is my number bigger or smaller than 2^4?

Reminder: argument must be positive

Do they have the same base?

$x+3>0$ $\quad x>-3$ $\qquad 2x+1>0$

Example 4 Solve Inequalities with Logarithms on Each Side

Solve $\log_4 (x+3) > \log_4 (2x+1)$

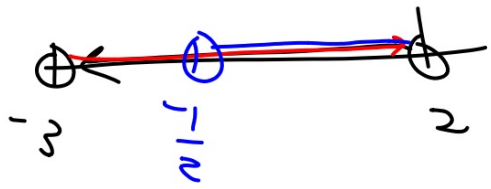
$2x>-1$ $\quad x>-\frac{1}{2}$

$(x+3) > (2x+1)$

$-x-1 \qquad -x-1$

$2 > x$

$-\frac{1}{2} < x < 2$



$x<2$

$2x+1>0$ $x+4>0$

$2x>-1$ $x>-4$

$x>-\frac{1}{2}$

GuidedPractice

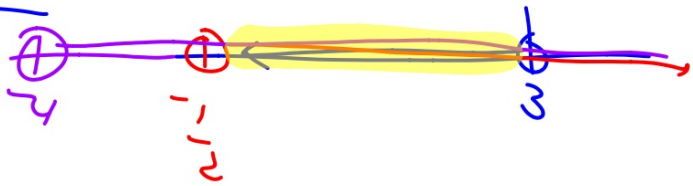
4. Solve $\log_5 (2x + 1) \leq \log_5 (x + 4)$. Check your solution.

$2x+1 \leq x+4$

$-x \quad -1 \qquad -x \quad -1$

$x \leq 3$

$-\frac{1}{2} < x < 3$



7.4 9-33 odd