Algebra 2 7.8

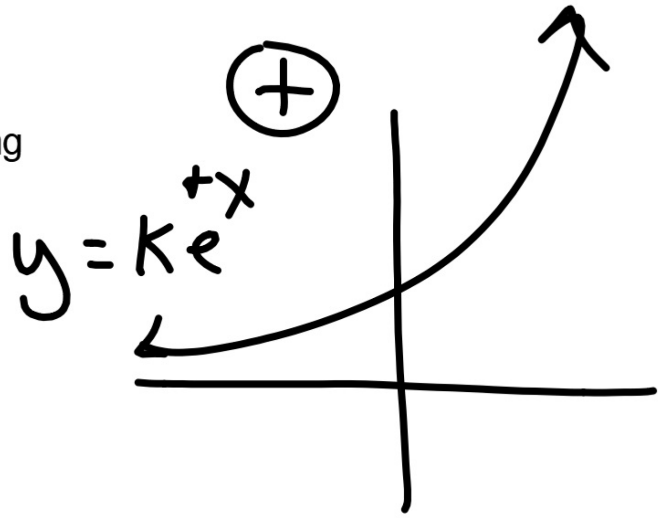
Use logarithms to solve problems involving exponential growth and decay

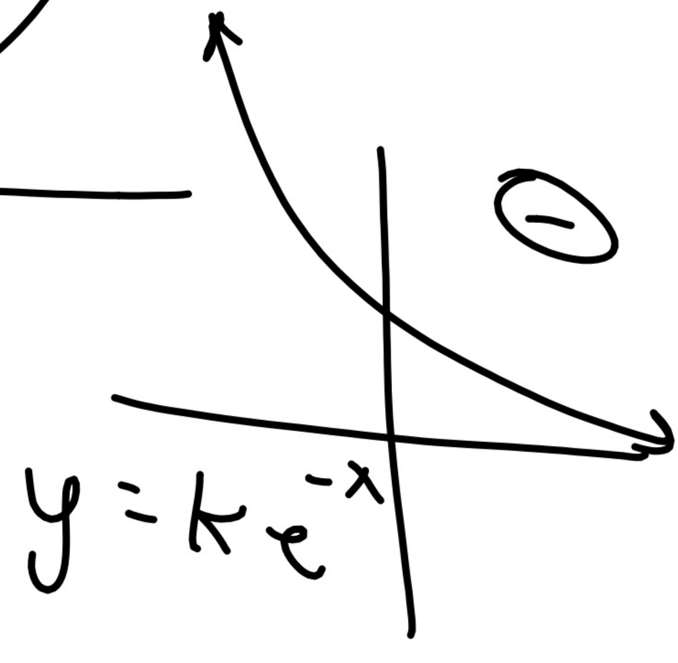
Use logarithms to solve problems of logistic growth

logistic

half-life

carbon dating





$+K \uparrow$ $-K \downarrow$

The exponential growth equation $y = ae^{kt}$ is identical to the continuously compounded interest formula you learned in Lesson 7-7.

Continuous Compounding

$$\boxed{\cancel{A = Pe^{rt}}}$$

a

P = initial amount
A = amount at time t
r = interest rate

Population Growth

$$\boxed{y = ae^{kt}}$$

a

a = initial population
y = population at time t
k = rate of continuous growth

9,360,000 $y = 8.18e^{0.02t}$

Real-World Example 3 Continuous Exponential Growth

POPULATION In 2007, the population of the state of Georgia was 9.36 million people. In 2000, it was 8.18 million.

a. Determine the value of *k*, Georgia's relative rate of growth. 0.02

(2007, 9.36)

(2000, 8.18)

$y = ae^{kt}$

$\frac{9.36}{8.18} = \frac{8.18}{8.18} e^{7k}$

$\ln 1.144 = \cancel{\ln} e^{7k}$

$0.135 = 7k$

$0.01925 = k$

$0.019 = k$

$0.02 = k$

b. When will Georgia's population reach 12 million people?

$$y = 8.18e^{0.02t}$$

$$12 = 8.18e^{0.02t}$$

$$\frac{12}{8.18} = \frac{8.18e^{0.02t}}{8.18}$$

$$\ln 1.467 = \cancel{\ln e}^{0.02t}$$

$$0.3832 = 0.02t$$

$$t = 19.2 \text{ yrs}$$

$1650 = 1000e^{k \cdot 40}$

$\frac{1650}{1000} = \frac{1000}{1000}e^{k \cdot 40}$

$\ln 1.65 = \cancel{\ln e}^{40k}$

$0.5008 = 40k$

$0.0125 = k$

Guided Practice

3. **BIOLOGY** A type of bacteria is growing exponentially according to the model $\boxed{y = 1000e^{kt}}$, where t is the time in minutes.

A. If there are 1000 cells initially and 1650 cells after 40 minutes, find the value of k for the bacteria.

(1650, 40)

B. Suppose a second type of bacteria is growing exponentially according to the model $y = 50e^{0.0432t}$. Determine how long it will be before the number of cells of this bacteria exceed the number of cells in the other bacteria.

$y = 1000e^{0.0125t}$ A

B>A

$y = 50e^{0.0432t}$ B more

A $\quad y = 1000e^{0.0125t}$

B $\quad y = 50e^{0.0432t}$

time B > A

$$\frac{50e^{0.0432t}}{50} > \frac{1000e^{0.0125t}}{50}$$

$$\cancel{\ln}\left(e^{0.0432t}\right) > \ln\left(20e^{0.0125t}\right)$$

$$0.0432t > 2.9957 + 0.0125t\,\cancel{\ln e}$$

$$-0.0125t \qquad\qquad -0.0125t$$

$$0.0307t > 2.9957$$

$$t > 97.6 \text{ min}$$

(increase/decrease)

KeyConcept Exponential Growth and Decay

Exponential Growth	Exponential Decay
Exponential growth can be modeled by the function $$f(x) = ae^{kt},$$ where a is the initial value, t is time in years, and k is a constant representing the **rate of continuous growth.**	Exponential decay can be modeled by the function $$f(x) = ae^{-kt},$$ where a is the initial value, t is time in years, and k is a constant representing the **rate of continuous decay.**

half-life = length of TIME
If you start with 100%...

$50 = 100e^{kT}$

Real-World Example 1 Exponential Decay

SCIENCE **The half-life of a radioactive substance is the time it takes for half of the atoms of the substance to disintegrate. The half-life of Carbon-14 is 5730 years. Determine the value of *k* and the equation of decay for Carbon-14.**

$\frac{50}{100} = \frac{100e^{k \cdot 5730}}{100}$

$\ln 0.5 = \ln e^{5730k}$

$-0.6931 = 5730k$

$-0.00012 = k$

$\boxed{y = ae^{-0.00012t}}$

GuidedPractice

1. The half-life of Plutonium-239 is 24,000 years. Determine the value of k.

$$\frac{50}{100} = \frac{100e^{k \cdot 24000}}{100}$$

$$\ln(.5) = \ln e^{24000k}$$

$$-0.6931 = 24000k$$

$$K = -.000029$$

$$y = ae^{-0.000029t}$$

So if it was 100% then...

Real-World Example 2 Carbon Dating

SCIENCE A paleontologist examining the bones of a prehistoric animal estimates that they contain 2% as much Carbon-14 as they would have contained when the animal was alive.

a. How long ago did the animal live? (Use k=-.00012 from ex. 1)

$2 = 100e^{-0.00012t}$

$\ln 0.02 = \ln e^{-0.00012t}$

$-391 = -0.00012t$

$t = 32{,}600$ yrs

Real-WorldLink

The oldest modern human fossil, found in Ethiopia, is approximately 160,000 years old.

Source: National Public Radio

b. **If prior research points to the animal being around 20,000 years old, how much Carbon-14 should be in the animal?**

$$x = 100e^{(20000(-0.00012))}$$

$$= 100e^{-2.4}$$

$$= 9\%$$

Study Tip

Carbon Dating When given a percent or fraction of decay, use an original amount of 1 for *a*. (100%)