common log = 10 base

Algebra 2 7.7 e ln

Evaluate expressions involving the natural base and natural logarithms

Solve exponential equations and inequalities using logarithms

common logarithm 10

irrational number

e ≈ 2.72...

exponential growth

exponential decay

natural logarithm

ln

whiteboards

e

log e

ln

$\log_3 27 = 3$

$\log_2 16 = 4$

$\log 100 = 2$

1 Base *e* and Natural Logarithms Like π and $\sqrt{2}$, the number e is an irrational number. The value of e is 2.71828… . It is referred to as the **natural base, *e***. An exponential function with base e is called a **natural base exponential function**.

$\log_{10} 100 = 2 \quad 10^2 = 100$

$\log_e = \ln_e = \ln$

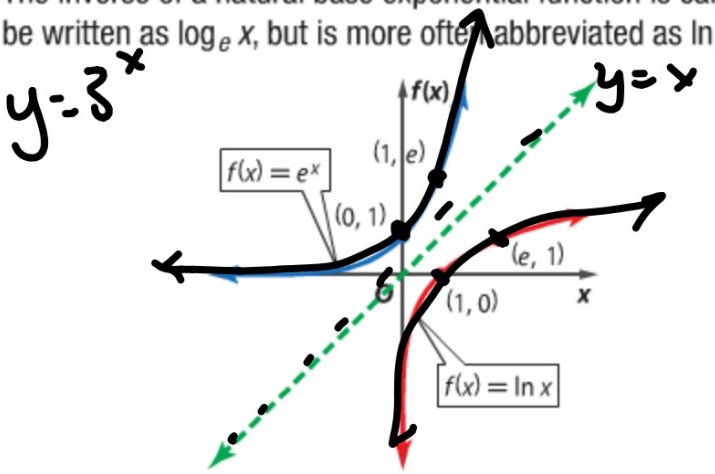
What's "natural" about it? Long story...
Napier
Mathematical simplicity (calculus)
Describes continuous growth of things in nature (populations, bacteria, etc.)

Why LN? (Latin) logarithmus naturalis

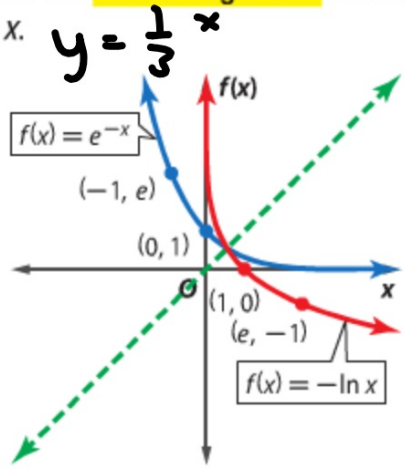
KeyConcept Natural Base Functions

The function $f(x) = e^x$ is used to model continuous exponential growth.
The function $f(x) = e^{-x}$ is used to model continuous exponential decay.

The inverse of a natural base exponential function is called the natural logarithm. This logarithm can be written as $\log_e x$, but is more often abbreviated as ln x.



Exponential Growth



Exponential Decay

$\ln 4 = x \rightarrow \log_e 4 = x \rightarrow e^x = 4$

$e^x = 4$

Example 1 Write Equivalent Expressions

Write each exponential equation in logarithmic form.

a. $e^x = 8$

$\log_e 8 = x$

$\ln 8 = x$

b. $e^5 = x$

$\log_e x = 5$

$\ln x = 5$

ln

$\log_e$

GuidedPractice

1A. $e^x = 9$

$\log_e 9 = x$

$\ln 9 = x$

1B. $e^7 = x$

$\log_e x = 7$

$\ln x = 7$

$e^x = 12 \quad \rightarrow \quad \log_e 12 = x \quad \rightarrow \quad \ln 12 = x$

Example 2 Write Equivalent Expressions

Write each logarithmic equation in exponential form.

a. $\ln x \approx 0.7741$ **b.** $\ln 10 = x$

$e^{0.7741} = x$ $e^{x} = 10$

GuidedPractice

2A. $\ln x \approx 2.1438$

$e^{2.1438} = x$

2B. $\ln 18 = x$

$e^x = 18$

Same rules as all other log expressions...
When we are adding exponents:
When we are subtracting exponents:
When we are multiplying exponents:

Example 3 Simplify Expressions with *e* and the Natural Log

Write each expression as a single logarithm.

a. $3 \cdot \ln 10 - \ln 8$

$3 \ln 10^3 - \ln 8$

$\ln \frac{10^3}{8} = \ln\left(\frac{1000}{8}\right)$

$= \ln(125)$

Just re-write:
Answer still uses ln( )
Don't try to solve...

Re-write ln( )

b. $\ln 40 + 2\ln\left(\frac{1}{2}\right) + \ln x$ → $\frac{1}{2}\cdot\frac{1}{2}$

$\ln 40 + \ln \frac{1}{4} + \ln x$

$\ln\left(40 \cdot \frac{1}{4} \cdot x\right)$

$\ln_e(10x)$

Rewrite ln( )

Guided Practice

3A. $6 \ln 8 - 2 \ln 4$

$\ln 8^6 - \ln 4^2$

$\ln 262144 - \ln 16$

$\ln(16384)$

3B. $2 \ln 5 + 4 \ln 2 + \ln 5y$

$\ln 5^2 + \ln 2^4 + \ln 5y$

$\ln(25 \cdot 16 \cdot 5y)$

$\ln(2000y)$

$e \longleftrightarrow \ln$

Because the natural base and natural log are inverse functions, they can be used to *undo* or eliminate each other.

$e(\ln)$ $\ln(e)$

(write in exp form)

$\log_{10} 10 = 1$ $10^1 = 10$

$\ln e = 1.$ $e^1 = e$

$\ln e^x = \ln 42.1$

$x \ln e = \ln 42.1$

$x = 3.7400$

$\ln_e(e^7) = 7$

$e^7 = e^x$

$7 = x$

$4e^x + 2 = 180$

$-2 \quad -2$

$4e^x = 178$

$\frac{4e^x}{4} = \frac{178}{4}$

$\ln(e^x) = \ln(44.5)$

$x =$

$x(\cancel{\ln e}) = 3.7955$

$x = 3.7955$