Algebra 2 5.5
Factor polynomials
Solve polynomial equations by factoring (Ch. 4 quadratics)
Re-write expressions in "quadratic form"...tough but necessary

x-factor (ch.4)
difference of 2 squares
sum of 2 cubes
difference of 2 cubes
prime polynomial
quadratic form

KeyConcept Sum and Difference of Cubes

Factoring Technique	General Case
Sum of Two Cubes	$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$
Difference of Two Cubes	$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

ConceptSummary Factoring Techniques

Number of Terms	Factoring Technique	General Case
any number	Greatest Common Factor (GCF)	$4a^3b^2 - 8ab = 4ab(a^2b - 2)$
two	Difference of Two Squares	$a^2 - b^2 = (a + b)(a - b)$
	Sum of Two Cubes	$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$
	Difference of Two Cubes	$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$
three	Perfect Square Trinomials	$a^2 + 2ab + b^2 = (a + b)^2$ $a^2 - 2ab + b^2 = (a - b)^2$
	General Trinomials	$acx^2 + (ad + bc)x + bd$ $= (ax + b)(cx + d)$
four or more	Grouping	$ax + bx + ay + by$ $= x(a + b) + y(a + b)$ $= (a + b)(x + y)$

Factor or solve?

22. $\frac{a^8}{a^2} - \frac{a^2b^6}{a^2}$

$$a^2(a^6 - b^6)$$

$a^2 \quad b^2$

F $\quad$ S

$$a^2(a^2 - b^2)(a^4 + a^2b^2 + b^4)$$

$$a^2(a+b)(a-b)(a^4 + a^2b^2 + b^4)$$

Factor or solve?

34. $x^3 + 216 = 0$

$x \quad 6$

$(x+6)(x^2-6x+36) = 0$

$x+6=0$

$x=-6$

$x^2-6x+36=0$

$x = \frac{6 \pm \sqrt{36-4\cdot1\cdot36}}{2}$

$= \frac{6 \pm \sqrt{-108}}{2} = \frac{6}{2} \pm \frac{6i\sqrt{3}}{2}$

$= 3 \pm 3i\sqrt{3}$

108

54 2

6 9

2 3 3 3

$3\cdot2\sqrt{3}$

$6i\sqrt{3}$

35. $64x^3 + 1 = 0$

49. $x^4 - 625$

$x^2 \quad 25$

$(x^2+25)(x^2-25)$

$(x^2+25)(x+5)(x-5)$

$\star\ \boxed{u = x^2}$

$x^4 - 29x^2 + 100$

$\boxed{(x^2)^2 - 29(x^2) + 100}$

$\star\ u^2 - 29u + 100$

$(x.x)(x.x)$ $\quad$ 29.x.x

$(x^2)^2$

Look at the first term...Can I write it using $(\quad)^2$?

Can I use the ( ) to re-write the middle term too?

$u = 4u^2$

$16x^4 - 20x^2 + 6$

$$\boxed{(4x^2)^2 - 5(4x^2) + 6}$$

$16(x^2)^2$

$$\boxed{\begin{array}{l} u^2 - 5u + 6 \\ u = 4\cancel{x}^2 \end{array}}$$

$u = n^4$

Example 5 Quadratic Form

Write each expression in quadratic form, if possible.

a. $150n^8 + 40n^4 - 15$

$150(n^4)^2 + 40(n^4) - 15$

$5(30(n^4)^2 + 8(n^4) - 3)$

Don't usually take out GCF for these...

Might need to leave it as a coefficient...

Sometimes there are options (better or not???)

Guided Practice

6A. $4x^4 - 8x^2 + 3 = 0$

$\boxed{u = 2x^2}$

$u = x^2$

$4u^2 - 8u + 3 = 0$

Re-write in quadratic form:
"u substitution"
Let u =

$(2x^2)^2 - 4(2x^2) + 3 = 0$

1. quad. form
2. u subs.
3. factor
4. solve u =
5. u = placeholder
6. simplify

$u^2 - 4u + 3 = 0$

$(u-1)(u-3) = 0$

3, -1, -3, -4

$u - 1 = 0$, $u = 1$

$u - 3 = 0$, $u = 3$

$\frac{2x^2}{2} = \frac{1}{2}$

$\sqrt{x^2} = \sqrt{\frac{1}{2}}$

$x = \pm\sqrt{\frac{1}{2}}$

$\frac{2x^2}{2} = \frac{3}{2}$

$\sqrt{x^2} = \sqrt{\frac{3}{2}}$

$x = \sqrt{\frac{1}{2}} = \frac{\sqrt{1}}{\sqrt{2}}\left(\frac{\sqrt{2}}{\sqrt{2}}\right) = \frac{\sqrt{2}}{2}$

$x = -\sqrt{\frac{1}{2}} = -\frac{\sqrt{2}}{2}$

$x = \sqrt{\frac{3}{2}} = \frac{\sqrt{3}}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{6}}{2}$

$x = -\sqrt{\frac{3}{2}} = -\frac{\sqrt{6}}{2}$

U substitution

Let u = $u = 3x^2$

Example 6 Solve Equations in Quadratic Form

Solve $18x^4 - 21x^2 + 3 = 0$.

$2(3x^2)^2 - 7(3x^2) + 3 = 0$

$2u^2 - 7u + 3 = 0$

$\frac{6}{\substack{1\ 6 \\ 2\ 3}}$ $(2u^2 - 6u)(-u + 3) = 0$

$2u(u-3) - 1(u-3) = 0$

$(u-3)(2u-1) = 0$

$u - 3 = 0$ $\qquad$ $2u - 1 = 0$

$u = 3$ $\qquad$ $2u = 1$

$u = \frac{1}{2}$

$\frac{3x^2}{3} = \frac{3}{3}$ $\qquad$ $\frac{3x^2}{3} = \frac{\frac{1}{2}}{3}$

$\sqrt{x^2} = \sqrt{1}$ $\qquad$ $\sqrt{x^2} = \sqrt{\frac{1}{6}}$

$x = \pm$ $\qquad$ $x = \pm\sqrt{\frac{1}{6}}$

$x = 1$

$x = -1$

$x = \sqrt{\frac{1}{6}} = \frac{\sqrt{1}}{\sqrt{6}} \cdot \frac{\sqrt{6}}{\sqrt{6}} = \frac{\sqrt{6}}{6}$

$x = -\sqrt{\frac{1}{6}} \cdot \frac{\sqrt{6}}{\sqrt{6}} = -\frac{\sqrt{6}}{6}$

Solve $u = 2x^2$

$$8x^4 + 10x^2 - 12 = 0$$

$$2(2x^2)^2 + 5(2x^2) - 12 = 0$$

4

$$2u^2 + 5u - 12 = 0$$

-24

1 24
2 12
-3 +8
4 6

$$(2u^2 + 8u) - 3u - 12 = 0$$

$$2u(u+4) - 3(u+4) = 0$$

$$(u+4)(2u-3) = 0$$

$u + 4 = 0$

$u = -4$

$$\frac{2x^2}{2} = \frac{-4}{2}$$

$$\sqrt{x^2} = \sqrt{-2}$$

$x = \sqrt{2}i$

$x = -\sqrt{2}i$

$x = \frac{\sqrt{3}}{2}$

$x = \pm\frac{\sqrt{3}}{2}$

$2u - 3 = 0$

$u = \frac{3}{2}$

$$2x^2 = \frac{3}{2}$$

$$\sqrt{x^2} = \sqrt{\frac{3}{4}}$$

$$x = \pm\frac{\sqrt{3}}{2}$$