Algebra 2 5.8

Identify possible rational* zeros of a polynomial function

Find all possible rational* zeros of a function

rational

zero (of a function)

leading coefficient (q)

constant (p)

* could be a fraction (but not $\sqrt{}$ or i)

LC C ± factors C / factors LC

6 −40

KeyConcept Rational Zero Theorem

Words If $P(x)$ is a polynomial function with integral coefficients, then every rational zero of $P(x) = 0$ is of the form $\frac{p}{q}$, a rational number in simplest form, where p is a factor of the constant term and q is a factor of the leading coefficient.

Example Let $f(x) = 6x^4 + 22x^3 + 11x^2 - 80x - 40$. If $\frac{4}{3}$ is a zero of $f(x)$, then 4 is a factor of -40, and 3 is a factor of 6.

Corollary to the Rational Zero Theorem

If $P(x)$ is a polynomial function with integral coefficients, a leading coefficient of 1, and a nonzero constant term, then any rational zeros of $P(x)$ must be factors of the constant term.

Try the factors of...

Example 1 Identify Possible Zeros

List all of the possible rational zeros of each function.

a. $f(x) = 4x^5 + x^4 + 2x^3 - 5x^2 + 8x + 16$

$\frac{\pm \text{factors of c}}{\text{factors of LC}}$

pos 2, 0

neg 3, 1

$\frac{1}{1}$ $\frac{1}{2}$ $\frac{1}{4}$ $\frac{2}{1}$ ~~$\frac{2}{2}$~~ ~~$\frac{4}{4}$~~ $\frac{4}{1}$ $\frac{8}{1}$ $\frac{16}{1}$

$\pm 1 \quad \pm\frac{1}{2} \quad \pm\frac{1}{4} \quad \pm 2 \quad \pm 4 \quad \pm 8 \quad \pm 16$

b. $f(x) = x^3 - 2x^2 + 5x + 12$

$\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12$

$(x + 3)$

$x =$

$x =$

$x =$

+ 2, 0

− 1

1	−2	5	12
	−1	3	−8
1	−3	8	4

GuidedPractice

1A. $g(x) = 3x^3 - 4x + 10$

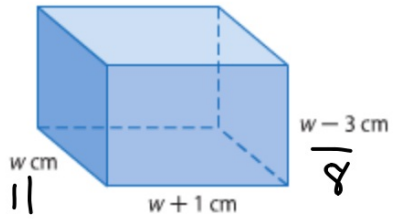
$$\frac{\pm 1, 2, 5, 10}{1, 3}$$

$$\pm\frac{1}{1} \quad \pm\frac{1}{3}, \quad \pm\frac{2}{1} \quad \pm\frac{2}{3} \quad \pm 5 \quad \pm\frac{5}{3} \quad \pm\frac{10}{1} \quad \pm\frac{10}{3}$$

1B. $h(x) = x^3 + 11x^2 + 24$

GuidedPractice $V = (\quad)(\quad)(\quad)$

2. GEOMETRY The volume of a rectangular prism is 1056 cubic centimeters. The length is 1 centimeter more than the width, and the height is 3 centimeters less than the width. Find the dimensions of the prism.



11 12 8

$1056 = (w)(w+1)(w-3)$

$1056 = w(w^2 - 2w - 3)$

-1056

$0 = w^3 - 2w^2 - 3w - 1056$

11 | 1 -2 -3 -1056
↓ 11 99 1056
1 9 96 0

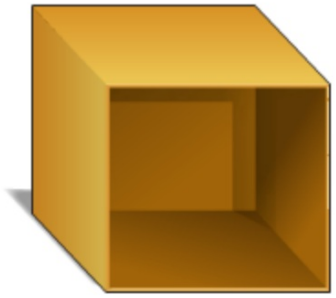
(+) 1

1	1056		
6	176	8	132
12	88	11	96

$w = 11$

Real-World Example 2 Find Rational Zeros

WOODWORKING Adam is building a computer desk with a separate compartment for the computer. The compartment for the computer is a rectangular prism and will be 8019 cubic inches. The compartment will be 24 inches longer than it is wide and the height will be 18 inches greater than the width. Find the dimensions of the computer compartment.



$\pm \frac{1, 3, 5, 15}{1, 2, 4}$

all rational

all real

$\pm$ ( )

(9) $f(x) = 4x^4 - 12x^3 + 25x^2 - 14x - 15$

(+) 3, 1

(−) 1

$\pm 1, \pm 3, \pm 5, \pm 15, \pm \frac{1}{2}, \pm \frac{3}{2}, \pm \frac{5}{2}, \pm \frac{15}{2}$

$\pm \frac{1}{4}, \pm \frac{3}{4}, \pm \frac{5}{4}, \pm \frac{15}{4}$ (+) 3, 1

$-\frac{1}{2}$ | 4 −12 25 −14 −15

↓ −2 7 −16 15

4 −14 32 −30 | 0

$x = 2$

$\frac{3}{2}$ | 4 −14 32 −30

↓ 6 −12 30

4 −8 20 | 0

$\frac{4x^2}{4} - \frac{8x}{4} + \frac{20}{4} = \frac{0}{4}$

$x^2 - 2x + 5 = 0$

$x = -\frac{1}{2}$

$x = \frac{3}{2}$

$x = 1 + 2i$

$x = 1 - 2i$

1 2 3 5 6 10 15 30

10

1, 2, 4