Algebra 2 5.7

Determine the number and type of roots for a polynomial equation

Find the zeros of a polynomial function *Ch. 4

degree (of an equation)

zero —

factor (x) Same x =

root —

x-intercept —

Fundamental theorem of algebra degree = no. of ans.

Descarte's rule of signs

complex number* 2 + i 2 - i

conjugate pair*

Quiz 5.5-5.6 Fri.

GuidedPractice

2. State the possible number of positive real zeros, negative real zeros, and imaginary zeros of $h(x) = 2x^5 + x^4 + 3x^3 - 4x^2 + x + 9$.

+ 2, 0

− 3, 1

2, 0; 3, 1; 0, 2, 4

r n i

Number of Positive Real Zeros	Number of Negative Real Zeros	Number of Imaginary Zeros	Total Number of Zeros
2	3	0	5
2	1	2	5
0	3	2	5
0	1	4	5

(Roots...) $\pm 1, 2, 4, 5, 8, 10 \ldots$

Example 3 Use Synthetic Substitution to Find Zeros

Find all of the zeros of $f(x) = x^4 - 18x^2 + 12x + 80$.

Try the factors of c...
trust me :)

$x = -4$

$x = -2$

$x = 3 + i$

$x = 3 - i$

$\oplus$ 2, 0 − 4

$\ominus$ 2, 0

$$\begin{array}{r|rrrrr} -4 & 1 & 0 & -18 & 12 & 80 \\ & \downarrow & -4 & 16 & 8 & -80 \\ \hline & 1 & -4 & -2 & 20 & 0 \end{array}$$

$$\begin{array}{r|rrrr} -2 & 1 & -4 & -2 & 20 \\ & \downarrow & -2 & 12 & -20 \\ \hline & 1 & -6 & 10 & 0 \end{array}$$

$x^2 - 6x + 10 = 0$

$$x = \frac{6 \pm \sqrt{36 - 4 \cdot 1 \cdot 10}}{2}$$

$$x = \frac{6 \pm \sqrt{-4}}{2} = \frac{6}{2} \pm \frac{2i}{2}$$

$3 \pm i$

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3. Find all of the zeros of $h(x) = x^3 + 2x^2 + 9x + 18$.

imag>>squiggles... always cong pairs

Review Vocabulary

complex conjugates two complex numbers of the form $a + bi$ and $a - bi$

In Chapter 4, you learned that the product of complex conjugates is always a real number and that complex roots always come in conjugate pairs. For example, if one root of $x^2 - 8x + 52 = 0$ is $4 + 6i$, then the other root is $4 - 6i$.

This applies to the zeros of polynomial functions as well. For any polynomial function with real coefficients, if an imaginary number is a zero of that function, its conjugate is also a zero. This is called the **Complex Conjugates Theorem**.

This is why the number of roots decreases by two every time...

KeyConcept Complex Conjugates Theorem

Words	Let a and b be real numbers, and $b \neq 0$. If $a + b\boldsymbol{i}$ is a zero of a polynomial function with real coefficients, then $a - b\boldsymbol{i}$ is also a zero of the function.
Example	If $3 + 4\boldsymbol{i}$ is a zero of $f(x) = x^3 - 4x^2 + 13x + 50$, then $3 - 4\boldsymbol{i}$ is also a zero of the function.

Write an equation with solutions of x= 3 and x= -2

$\boxed{x^2 - x - 6 = 0}$

$x^2 - 3x + 2x - 6 = 0$

$(x-3)(x+2) = 0$

$(x-3) = 0 \qquad (x+2) = 0$

$x = 3 \qquad x = -2$

Write an equation with a solution of x= 2i

$\boxed{x^2 + 4 = 0}$

reminder:
equations have to = something...

$(x-2i)(x+2i) = 0$

$(x-2i) = 0 \qquad (x+2i) = 0$

$x = 2i \qquad x = -2i$

$x - 2i$

$x + 2i$

$2ix - 4ii$

$x^2 - 2ix$

$x^2 + 4$

reverse steps

pair

Write an equation with a solution of x = 3 + 2i

$x^2 - 6x + 13 = 0$

$x - 3 - 2i$

$x - 3 + 2i$

$\cancel{-2ix}\ \cancel{-6i}\ -4i^2$ $+4$

$-3x + 9\ \cancel{+6i}$

$x^2\ -3x\ \cancel{-2ix}$

$x^2 - 6x + 13$

$(x-3-2i)(x-3+2i) = 0$

$(x-3-2i) = 0 \qquad (x-3-2i)$

$x = 3+2i \qquad x = 3-2i$

$x^2 + \frac{1}{2}x - \frac{5}{2} = 0 \Rightarrow 2x^2 + x - 5 = 0$

Example 4 Use Zeros to Write a Polynomial Function

Write a polynomial function of least degree with integral coefficients, the zeros of which include -1 and $5 - i$.

$= 0$

$x^3 - 9x^2 + 16x + 26 = 0$

$(x+1)(x^2 - 10x + 26) = 0$

$(x+1)(x - 5 + i)(x - 5 - i) = 0$

$x = -1 \quad x = 5 - i \quad x = 5 + i$

$-5i - 5 + i = 0$

$x - 5 + i$
$x - 5 + -i$

$-i\cancel{x} + \cancel{5}i - i \cdot i$
$-5x + 25 - \cancel{5}i$
$x^2 - 5x + \cancel{xi}$

$x^2 - 10x + 26$

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4. Write a polynomial function of least degree with integral coefficients having zeros that include -1 and $1 + 2i$.

1–17 odd 19–24 all

$x^2 - 5$

$x + 1$

$$x^3 \quad x^2 - 5$$
$$x^3 \quad -5x$$

$$x^3 + x^2 - 5x - 5 = 0$$

$$(x+1)\underbrace{(x^2 - 5)}_{} = 0$$

$$(x+1)(x+\sqrt{5})(x-\sqrt{5}) = 0$$

$x = -1 \qquad x = -\sqrt{5} \qquad x = \sqrt{5}$

$$x + \sqrt{5}$$
$$x - \sqrt{5}$$
$$x^2 + \sqrt{5}x - 5$$
$$-\sqrt{5}x$$
$$x^2 - 5$$