Algebra 2 6.6 $(\quad)^{\frac{1}{2}}$ $\sqrt{\quad}$

Write expressions with rational exponents in radical form and vice versa*

*Alg 1 7.3

Simplify expressions in exponential and radical form

radical form

rational exponents

whiteboards

$b^{\frac{1}{2}} = \sqrt{b}.$

$(x)^{\frac{1}{3}}$ $\sqrt[3]{x}$

KeyConcept

Words	For any real number b and any positive integer n, $b^{\frac{1}{n}} = \sqrt[n]{b}$, except when $b < 0$ and n is even. When $b < 0$ and n is even, a complex root may exist.
Examples	$27^{\frac{1}{3}} = \sqrt[3]{27}$ or 3 $\quad$ $(-16)^{\frac{1}{2}} = \sqrt{-16}$ or $4i$

Example 1 Radical and Exponential Forms

Simplify.

a. Write $x^{\frac{1}{6}}$ in radical form.

b. Write $\sqrt[4]{z}$ in exponential form.

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1A. Write $a^{\frac{1}{5}}$ in radical form.

$\sqrt[5]{a}$

1B. Write $\sqrt[8]{c}$ in exponential form.

$c^{\frac{1}{8}}$

1C. Write $d^{\frac{7}{4}}$ in radical form.

$\sqrt[4]{ddddddd}$

$d\sqrt[4]{d^3}$

1D. Write $\sqrt[3]{c^{-5}}$ in exponential form.

$(c^{-5})^{\frac{1}{3}}$

$c^{-\frac{5}{3}}$

powers...roots...reciprocals...

Example 2 Evaluate Expressions with Rational Exponents

Evaluate each expression.

a. $81^{\frac{1}{4}} = 3$

$(?)^4 = 81$

b. $216^{\frac{2}{3}} =$

$(216^2)^{\frac{1}{3}}$

$(46656)^{\frac{1}{3}}$

36

$*\ (216^{\frac{1}{3}})^2$

$(6)^2 = 36$

GuidedPractice

2A. $-3125^{-\frac{1}{5}}$

2B. $256^{\frac{3}{8}}$

$\left(-3125^{\frac{1}{5}}\right)^{-1}$

$(-5)^{-1}$

$-\frac{1}{5}$

$-(\quad)$

$\left(256^{\frac{1}{8}}\right)^{3} = \left(256^{\frac{1}{8}}\right)^{3}$

$(2)^{3}$

8

KeyConcept Rational Exponents

Words For any real nonzero number b, and any integers x and y, with $y > 1$, $b^{\frac{x}{y}} = \sqrt[y]{b^x} = \left(\sqrt[y]{b}\right)^x$, except when $b < 0$ and y is even. When $b < 0$ and y is even, a complex root may exist.

Examples $27^{\frac{2}{3}} = \left(\sqrt[3]{27}\right)^2 = 3^2$ or 9 $\qquad (-16)^{\frac{3}{2}} = \left(\sqrt{-16}\right)^3 = (4i)^3$ or $-64i$

Usually works better to do the root first

annual = one time per year

Real-World Example 3 Solve Equations with Rational Exponents

FINANCIAL LITERACY Refer to the beginning of the lesson. Suppose a video game system costs $390 now. How much would the price increase in six months with an annual inflation rate of 5.3%?

$\frac{1}{2}$ yr

$C = c(1 + r)^n$

final initial (+ rate)

$y = 390(1 + 0.053)^{\frac{1}{2}}$

n

$400.20

increase $10.20

GuidedPractice

3. Suppose a gallon of milk costs $2.99 now. How much would the price increase in 9 months with an annual inflation rate of 5.3%?

$C = c(1+r)^n$

$= 2.99(1.053)^{0.75}$

3.11

$0.12 increase

$x^3x^4 = ??$ x^7

$\sqrt{3} \cdot \sqrt{7} = \sqrt{21}$

$\sqrt[3]{15} \cdot \sqrt{3} =$

Example 4 Simplify Expressions with Rational Exponents

Simplify each expression.

a. $a^{\frac{2}{7}} \cdot a^{\frac{4}{7}} = a^{\frac{6}{7}}$

$a^{\frac{1}{2\cdot3}} \cdot a^{\frac{1}{2\cdot3}} = a^{\frac{2+3}{6}} = a^{\frac{5}{6}}$

$\sqrt[3]{a} \cdot \sqrt[2]{a}$

b. $b^{-\frac{5}{6}}$

$\frac{1}{b^{\frac{5}{6}}} = \frac{1}{\sqrt[6]{b^5}} \cdot \frac{\sqrt[6]{b}}{\sqrt[6]{b}}$

$\frac{\sqrt[6]{b}}{b}$

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4A. $p^{\frac{1}{4}} \cdot p^{\frac{9}{4}} = p^{\frac{10}{4}} = p^{\frac{5}{2}}$

$\sqrt[4]{p^{10}} \quad \sqrt[2]{p^5}$

$p^2\sqrt[4]{p^2} \quad p^2\sqrt{p}$

4B. $r^{-\frac{4}{5}} = \frac{1}{r^{\frac{4}{5}}} = \frac{1}{\sqrt[5]{r^4}}$

$= \frac{1}{\sqrt[5]{r^4}\sqrt[5]{r}} = \frac{1}{\sqrt[5]{r^4}} \cdot \frac{\sqrt[5]{r}}{\sqrt[5]{r}}$

$= \frac{\sqrt[5]{r}}{r}$

$\frac{3}{4} - \frac{1}{2}\cdot\frac{2}{2} = \frac{3-2}{4}$

Example 5 Simplify Radical Expressions

Simplify each expression.

a. $\frac{\sqrt[4]{3^3}}{\sqrt{3}} = \frac{3^{\frac{3}{4}}}{3^{\frac{1}{2}}} = 3^{\frac{1}{4}} = \sqrt[4]{3^1}$

b. $\sqrt[3]{64z^6}$ $4z^2$

p. 426

17-51 odd

not 39

$\frac{x^5}{x^3} = x^2$

Can you write all on the same base???
Might be easier to simplify if in rational form...
Use the smallest index possible

write in radical form?

c. $\dfrac{x^{\frac{1}{2}} - 2}{3x^{\frac{1}{2}} + 2}$

Wed

try rational expon?

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5A. $\frac{\sqrt[4]{32}}{\sqrt[3]{2}}$

5B. $\sqrt[3]{16x^4}$

ConceptSummary Expressions with Rational Exponents

An expression with rational exponents is simplified when all of the following conditions are met.

- It has no negative exponents.
- It has no exponents that are not positive integers in the denominator. (no radicals in denom)
- It is not a complex fraction.
- The index of any remaining radical is the least number possible. *