

Algebra 1 7.6

Solve problems involving exponential growth

Solve problems involving exponential decay

exponent

base

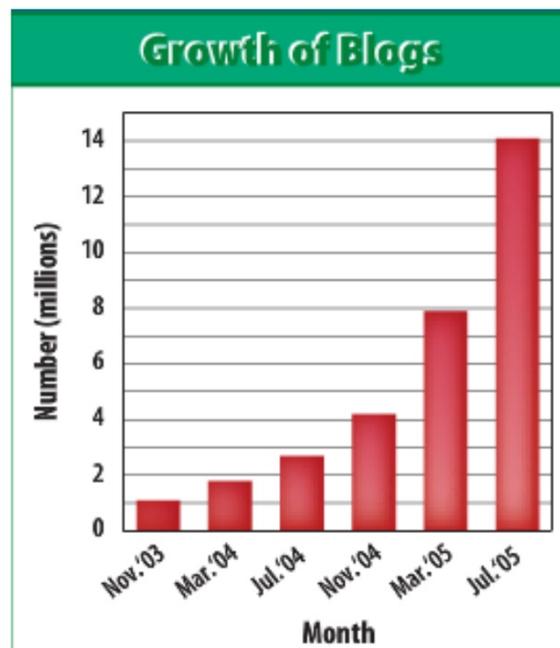
whiteboards

$$\begin{aligned} \$10,000 & \quad 0.06 \quad n=4 \\ t=18 \end{aligned}$$

$$\begin{aligned} A &= P \left(1 + \frac{r}{n}\right)^{n \cdot t} \\ &= 10,000 \left(1 + \frac{0.06}{4}\right)^{4 \cdot 18} \end{aligned}$$

$$10,000 (1.015)^{72}$$

$$\begin{aligned} & 2.92116 \\ \$ & 29,211.58 \end{aligned}$$



1 Exponential Growth The equation for the number of blogs is in the form $y = a(1 + r)^t$. This is the general equation for exponential growth.

KeyConcept Equation for Exponential Growth

a is the initial amount.

t is time.

$$y = a(1 + r)^t$$

y is the final amount.

r is the rate of change expressed as a decimal, $r > 0$.

What is the unit for time? (Can vary...need to know what it is for each problem)

6em4

Guided Practice

1. **TUITION** A college's tuition has risen 5% each year since 2000. If the tuition in 2000 was \$10,850, write an equation for the amount of the tuition t years after 2000. Predict the cost of tuition for this college in 2015.

$r = 0.05$
↑

$$y = 10,850(1 + 0.05)^t$$

1. Write equation (must have a variable)
2. Use the equation (answer the question)

$$y = 10,850(1 + 0.05)^{15}$$

$$10,850(1.05)^{15}$$

\$ 2,556.37

Compound interest is interest earned or paid on both the initial investment and previously earned interest. It is an application of exponential growth.

KeyConcept Equation for Compound Interest

The diagram shows the compound interest formula $A = P\left(1 + \frac{r}{n}\right)^{nt}$ with several annotations:

- A** is the current amount.
- P** is the principal or initial amount.
- r** is the annual interest rate expressed as a decimal, $r > 0$.
- n** is the number of times the interest is compounded each year, and **t** is time in years.
- Handwritten annotations: "orig. rate" with an arrow pointing to r ; "compound" with an arrow pointing to the fraction $\frac{r}{n}$; "nt" with an arrow pointing to the exponent.

— Will give this formula on quiz/test if needed (but you have to know what to do with it). Time is always in YEARS for CI.

If $n = 12$:

They get $1/12$ of their interest 12 times per year.

If $n = 4$:

They get $1/4$ of their interest 4 times per year.

etc.

52	weekly
12	monthly
1	annually
4	quarterly
	etc.
365	daily

Real-World Example 2 Compound Interest



FINANCE Maria's parents invested $\overset{P}{\$14,000}$ at $\overset{0.06}{6\%}$ per year compounded monthly.
How much money will there be in the account after 10 years? $n=12$

$$A = P \left(1 + \frac{r}{n}\right)^{nt}$$

$$= 14000 \left(1 + \frac{0.06}{12}\right)^{12 \cdot 10}$$

$$= 14000 (1.005)^{120}$$

$(14000) 1.8194$

\$ 25,471.55

Guided Practice

2. **FINANCE** Determine the amount of an investment if \$300 is invested at an interest rate of 3.5% compounded monthly for 22 years.

$$\begin{aligned} A &= P \left(1 + \frac{r}{n}\right)^{nt} \\ &= 300 \left(1 + \frac{0.035}{12}\right)^{12 \cdot 22} \\ &= 300 (1.0029166)^{264} \\ &= 300 \cdot 2.1573 \end{aligned} \quad \$647.20$$

depreciation

 **KeyConcept** Equation for Exponential Decay (decrease)

a is the initial amount.

t is time.

$$y = a(1 - r)^t$$

y is the final amount.

r is the rate of decay expressed as a decimal, $0 < r < 1$.

How is this formula different from the exponential increase formula?
Time unit can vary...

Equation for Exponential Growth

a is the initial amount.

t is time.

$$y = a(1 + r)^t$$

y is the final amount.

r is the rate of change expressed as a decimal, $r > 0$.

Equation for Exponential Decay

a is the initial amount.

t is time.

$$y = a(1 - r)^t$$

y is the final amount.

r is the rate of decay expressed as a decimal, $0 < r < 1$.

Is it getting more (growth) or less (decay)?

0.066

 **Real-World Example 3** Exponential Decay

PT

SWIMMING A fully inflated child's raft for a pool is losing 6.6% of its air every day. The raft originally contained 4500 cubic inches of air.

a. Write an equation to represent the loss of air.

Must have a variable!

$$y = a(1-r)^t$$
$$= 4500(1-0.066)^t = 4500(0.934)^t$$

b. Estimate the amount of air in the raft after 7 days.

$$= 4500(1-0.066)^7$$
$$= 2790$$

Is it increasing (+) or decreasing (-)?

p. 129

Guided Practice

3. **POPULATION** The population of Campbell County, Kentucky, has been decreasing at an average rate of about 0.3% per year. In 2000, its population was 88,647.
 (a) Write an equation to represent the population since 2000. If the trend continues,
 (b) predict the population in 2010.

.003



Equations need a variable!

(a) $y = a(1-r)^t$
 $y = 88647(1-0.003)^t$
(b) $y = 88647(1-0.003)^{10}$

86,023.2632

86,023

 **Real-World Example 1** Exponential Growth



CONTEST The prize for a radio station contest begins with a \$100 gift card. Once a day, a name is announced. The person has 15 minutes to call or the prize increases by 2.5% for the next day.

- a. Write an equation to represent the amount of the gift card in dollars after t days with no winners.

must have a variable