

$$\frac{n^5}{n^2}$$

$$x^3 \cdot x^2 =$$

Algebra 2 7.5

Simplify and evaluate expressions using the properties of logarithms
Solve logarithmic equations

logarithm = exponent

product rule

quotient rule

power^{power}

add expon.

subtract expon.

mult expon.

$$\star (x^3)^4$$

log=exponent so same rules apply

$$x^2 \cdot x^3$$

Key Concept Product Property of Logarithms

Words The logarithm of a product is the sum of the logarithms of its factors.

Symbols For all positive numbers a , b , and x , where $x \neq 1$, $\log_x ab = \log_x a + \log_x b$.

Example $\log_2 \underline{(5)(6)} = \log_2 \underline{(5)} + \log_2 \underline{(6)}$

$$x^2 \cdot x^3 = x^5$$

Example 3 Power Property of Logarithms

Given $\log_2 5 \approx 2.3219$, approximate the value of $\log_2 25$.

BOGO: $\log_2 2$

$$\log_2 2 = 1$$

Write as combination
of 2s & 5s
Check answer

$$2^{2.3219} = 5$$

$$2^1 = 2$$

$$\log_2(5 \cdot 5) =$$

$$2^? = 25$$

$$\log_2 5 + \log_2 5$$

$$2.3219 + 2.3219$$

$$2^{4.6438} = 25$$

$$= 4.6438$$

$$3 \cdot 64$$

$$3 \cdot 8 \cdot 8$$

$$3 \cdot 4 \cdot 4 \cdot 4$$

BOGO: $\log_4 4 =$

27
81
12

Example 1 Use the Product Property

Use $\log_4 3 \approx 0.7925$ to approximate the value of $\log_4 192$.

$$\log_4 4 = 1$$

$$\log_4 192 = \log_4 (3 \cdot 4 \cdot 4 \cdot 4) = \log_4 3 + \log_4 4 + \log_4 4 + \log_4 4$$

$$= 0.7925 + 1 + 1 + 1$$

$$= 3.7925$$

How can I write 192?

write using factors of:

3 (given) and 4 (base)

combine by adding exponents

check answer

Guided Practice

1. Use $\log_4 2 = 0.5$ to approximate the value of $\log_4 32$.

$$\log_4 4 = 1$$

BOGO: $\log_4 4$

write using factors of 2 and 4

(base)

$$\log_4 (2 \cdot 4 \cdot 4) = \log_4 2 + \log_4 4 + \log_4 4$$

$$\approx 0.5 + 1 + 1$$

$$\stackrel{2.5}{4} =$$

$$= 2.5$$

· **Guided Practice**

3. Given $\log_3 7 \approx 1.7712$, approximate the value of $\log_3 49$.

BOGO: $\log_3 3$

Write as combination of 3s & 7s
(base)

$$\begin{aligned}\log_3(7 \cdot 7) &= \log_3 7 + \log_3 7 \\ &= 1.7712 + 1.7712 \\ &= \end{aligned}$$

Log=exp so follow same rules x^5/x^2

$$\frac{x^5}{x^2}$$

KeyConcept Quotient Property of Logarithms

Words The logarithm of a quotient is the difference of the logarithms of the numerator and the denominator.

Symbols For all positive numbers a , b , and x , where $x \neq 1$,
 $\log_x \left(\frac{a}{b} \right) = \log_x a - \log_x b$.

Example $\log_2 \left(\frac{5}{6} \right) = \log_2 5 - \log_2 6$

Hint: divide out -1 to solve for log
Write in exp form

Real-World Example 2 Quotient Property

SCIENCE The pH of a substance is defined as the concentration of hydrogen ions $[H^+]$ in moles. It is given by the formula $pH = -\log_{10} H^+$. Find the amount of hydrogen in a liter of acid rain that has a pH of 4.2.

$$pH = -\log_{10} H^+$$

Note change in formula

Write in exp form

log=exp so follow same rules: $(x^2)^4$

 **KeyConcept** Power Property of Logarithms

Words The logarithm of a power is the product of the logarithm and the exponent.

Symbols For any real number p , and positive numbers m and b , where $b \neq 1$, $\log_b m^p = p \log_b m$.

Example $\log_2 6^5 = 5 \log_2 6$

hint: exponents mean repeated multiplication

$$\left. \begin{array}{l} \log_5 3 = 0.6826 \quad \log_5 4 = 0.8614 \\ \log_5 30 \quad \log_5 5 = 1 \end{array} \right\} \log_5 2 = 0.4307$$

$$\begin{array}{l} 30 \\ 5 \cdot 3 \cdot 2 \\ \quad 2.1133 \\ \quad \quad 5 \end{array} \quad \log_5 (5 \cdot 3 \cdot 2) = \log_5 5 + \log_5 3 + \log_5 2$$

$$= 1 + 0.6826 + 0.4307$$

$$= 2.1133$$

What does it mean if exponents are added?

X

bae

$$x = 12$$

1. mult
2. exp form
3. solve
4. check ans

Example 4 Solve Equations Using Properties of Logarithms

Solve $\log_6 x + \log_6 (x - 9) = 2$.

$$\log_6 \underline{x(x-9)} = 2$$

$$\log_6 (x^2 - 9x) = 2$$

$$6^2 = x^2 - 9x$$

$$36 = x^2 - 9x$$

$$0 = x^2 - 9x - 36$$

$$0 = (x - 12)(x + 3)$$

$$x = 12$$

$$x = -3$$

$$\begin{array}{r} -36 \\ -12 \quad +3 \\ -9 \end{array}$$

$$\log_6 4 + \log_6 9 = 2$$

$$4B. \log_6 x + \log_6 (x+5) = 2$$

$$\log_6 (x(x+5)) = 2$$

$$\log_6 (x^2 + 5x) = 2$$

$$36 = x^2 + 5x$$

$$0 = x^2 + 5x - 36$$

$$0 = (x+9)(x-4)$$

$$\cancel{x = 9}$$

$$x = 4$$

$$\begin{array}{r} -36 \\ 9 \times -4 \\ \hline 5 \end{array}$$

What does it mean if exponents are multiplied?

Guided Practice

4A. ~~$\log_7 x^2 = \log_7 27 + \log_7 3$~~

$$x^2 = (27 \cdot 3)$$

$$x^2 = 81$$

$$x = \pm 9$$

$$x = 9$$

$$(x^3)^2 = x^6$$

$$\frac{X^5}{X^3}$$

$$(X^2)^3$$

$$X^2 \cdot X^3$$

$$\log_7 100 - \log_7 (y+5) = \log_7 10$$

$$\cancel{\log_7 \left(\frac{100}{y+5} \right)} = \cancel{\log_7 10}$$

$$\frac{100}{y+5} = \frac{10}{1}$$

$$y = 5$$

$$10y + 50 = 100$$

$$10y = 50$$