Trig 3.8

Solve problems involving direct*, inverse**, joint** variation

direct variation $\quad\uparrow\ \uparrow\quad E = 2 \cdot P \qquad A = k \cdot B$

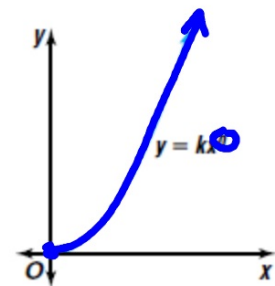inverse variation $\quad\uparrow\ \downarrow\quad A = \dfrac{k}{B}$

constant of variation $\quad k$

directly proportional



inversely proportional

joint variation (combination) $\quad A = \dfrac{k \cdot B \cdot C}{D}$

activity: whiteboards (if time)

k= constant

$(6, 27)$

1 Suppose $y$ varies directly as $x$ and $y = 27$ when $x = 6$.

a. Find the constant of variation and write an equation of the form $y = kx^n$

b. Use the equation to find the value of $y$ when $x = 10$.

a. 
$$y = kx$$

$$\frac{27}{6} = \frac{k \cdot 6}{6}$$

$$k = \frac{9}{2}$$

a. 
$$y = \frac{9}{2}x$$

b. 
$$y = \frac{9}{2} \cdot 10 = 45$$

**3** If $y$ varies directly as the cube of $x$ and $y = -67.5$ when $x = 3$, find $x$ when $y = -540$.

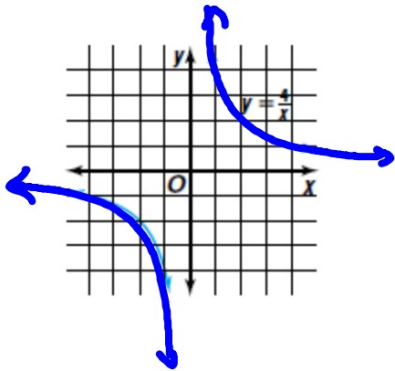$$y = kx^3$$

$$-67.5 = k \cdot 27$$

$$-2.5 = k$$

$$( 3, -67.5 )$$

$$y = -2.5x^3$$

$$-540 = -2.5x^3$$

$$216 = x^3$$

$$(216)^{\wedge\left(\frac{1}{3}\right)}$$

$$x = 6$$

$y = \frac{4}{x}$

inverse variation
driving
leftovers
homework

$$\overline{y = kx}$$

**4** If $y$ varies inversely as $x$ and $y = 21$ when $x = 15$, find $x$ when $y = 12$.

$(15, 21)$

$$y = \frac{k}{x}$$

$$\frac{21}{1} = \frac{k}{15}$$

$$k = 315$$

$$y = \frac{315}{x}$$

$$x \cdot 12 = \frac{315 \cdot x}{x}$$

$$\frac{12x}{12} = \frac{315}{12}$$

$$x = 26.25$$

**Joint Variation** — $y$ varies jointly as $x^n$ and $z^n$ if there is some nonzero constant $k$ such that $y = kx^nz^n$, where $x \neq 0$, $z \neq 0$, and $n > 0$.

**Example** ⑤ **GEOMETRY** The volume $V$ of a cone varies jointly as the height $h$ and the square of the radius $r$ of the base. Find the equation for the volume of a cone with height 6 centimeters and base diameter 10 centimeters that has a volume of $50\pi$ cubic centimeters.
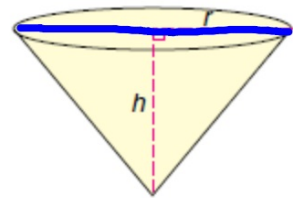
$V = k \cdot h \cdot r^2$

$80\pi = k \, 6 \cdot 5^2$

$\dfrac{50\pi}{150} = \dfrac{150k}{150}$

$k = \frac{1}{3}\pi$

$V = \frac{1}{3}\pi h r^2$

$V = \left(\frac{1}{3}\pi\right) r^2 h$

whiteboards

**Find the constant of variation for each relation and use it to write an equation 1 each statement. Then solve the equation.** $y = \frac{k}{x}$ $3 = \frac{k}{4}$ $y = \frac{12}{x}$

5. If $y$ varies inversely as $x$ and $y = 3$ when $x = 4$, find $y$ when $x = 15$.

6. If $y$ varies directly as the square of $x$ and $y = -54$ when $x = 9$, find $y$ when $x = 6$. $y = kx^2$ $-54 = k \cdot 81$ $y = -\frac{2}{3}x^2$

7. If $y$ varies jointly as $x$ and the cube of $z$ and $y = 16$ when $x = 4$ and $z = 2$, find $y$ when $x = -8$ and $z = -3$.

$$y = \frac{k \cdot x \cdot z^3}{\sqrt{n}}$$

and inv. as $\sqrt{n}$

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