Trig 3.7 $\quad \dfrac{2}{5} \qquad \dfrac{1}{x} \qquad$ Quiz 3.4-3.6

Graph rational functions
Determine horizontal, vertical, slant asymptotes

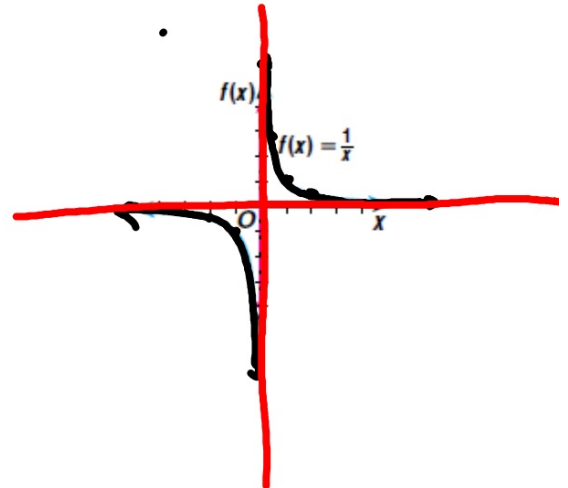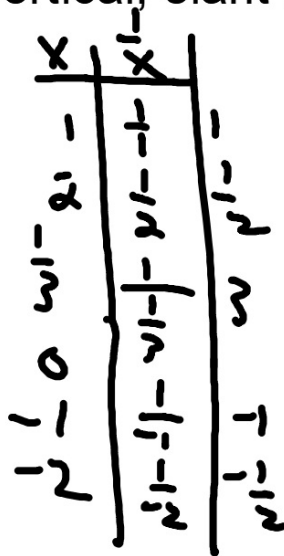rational function y=1/x

vertical asymptote
VA X=0
horizontal asymptote
HA y=0
point discontinuity

slant asymptote

common factor

x	$\frac{1}{x}$
1	1
2	$\frac{1}{2}$
3	$\frac{1}{3}$
0	$\frac{1}{2}$
-1	-1
-2	$-\frac{1}{2}$

$f(x)$ $f(x) = \frac{1}{x}$

Determine the equations of the vertical and horizontal asymptotes, if any, of each function.

5. $f(x) = \dfrac{x}{x^2 - 5}$ $y = \dfrac{x^1}{x^2} = 1$

6. $g(x) = \dfrac{x^3}{(x-2)(x+1)^2}$ $\dfrac{x^3}{x^2}$ $x - 2 = 0$

Slant

$x = 2$

VA $x = 5$

VA $x = 2$ $x = -1$ $x + 1 = 0$

$x = -1$

HA $y = 1$

HA none

Vertical asymptotes:
if denom =0

Horizontal asymptotes (degree):
- denom is higher y=0
- numerator = denominator y=their ratio
✗ denom is lower no HA
            (could be slant asym)

Slant asymptotes: (degree)
✗ numerator > denominator by exactly 1

Special: repeated factors
Is there a point discontinuity (hole)?

SA $y = x - 1$

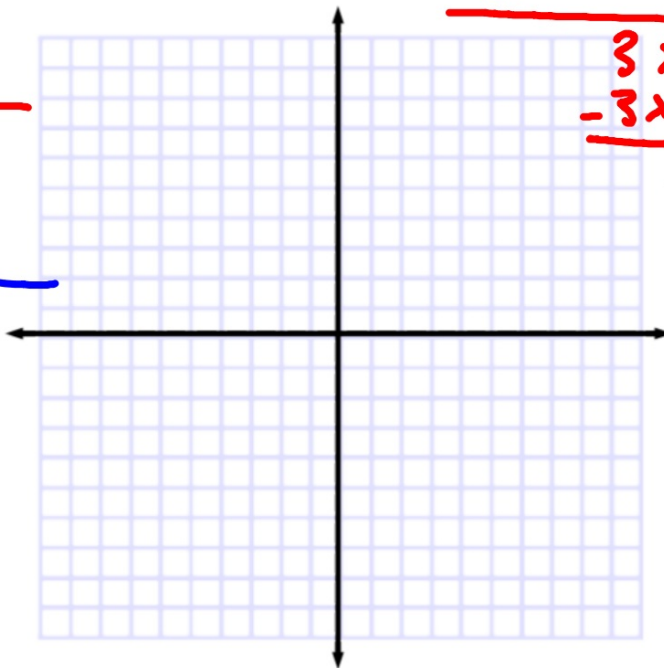Determine the slant asymptote of each function.

30. $f(x) = \dfrac{x^2 + 3x - 3}{x + 4}$

$$
\begin{array}{r}
x - 1 \\
x+4 \overline{\smash{\big)}\ x^2 + 3x - 3} \\
-x^2 + 4x \\
\hline
-x - 3 \\
+x + 4 \\
\hline
1
\end{array}
$$

31. $f(x) = \dfrac{x^2 + 3x - 4}{x}$

SA $y = x + 3$

$$
\begin{array}{r}
x + 3 \\
x \overline{\smash{\big)}\ x^2 + 3x - 4} \\
-x^2 \\
\hline
3x - 4 \\
-3x \\
\hline
-4
\end{array}
$$

There are times when the numerator and denominator of a rational function share a common factor. Consider $f(x) = \dfrac{(x + 2)(x - 3)}{x - 3}$. Since an x-value of 3 results in a denominator of 0, you might expect there to be a vertical asymptote at $x = 3$. However, $x - 3$ is a common factor of the numerator and denominator.
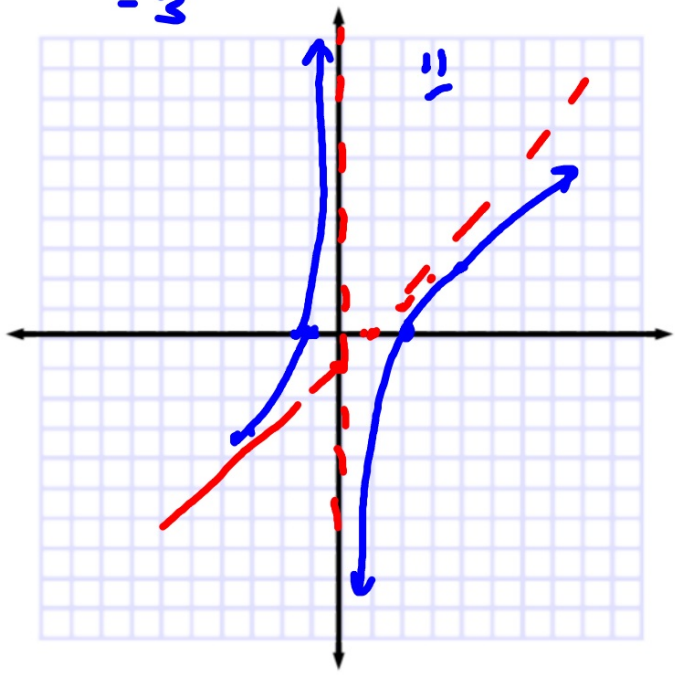
$y = x + 2$

$X - 3 = 0$

$X = 3$
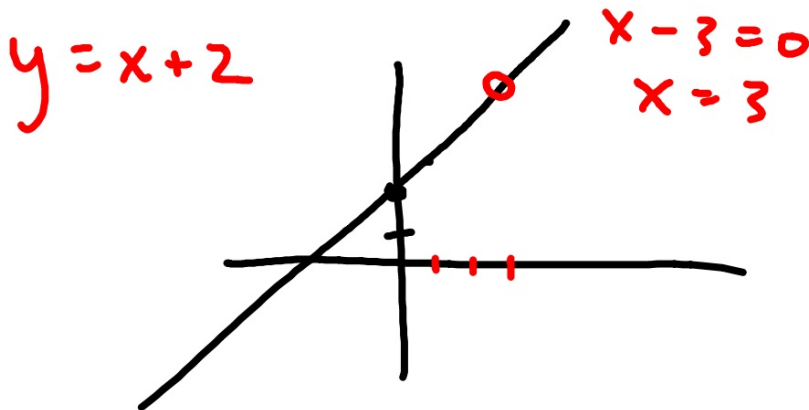
Graph each function.

$X-2=0$ $X=2$

$X+1=0$

35. $y = \dfrac{(x-2)(x+1)}{x}$

$\dfrac{X^2 - X - 2}{X}$ $X=-1$

$\dfrac{-5 \cdot -2}{-3}$

Determine:
common factors (holes)
VA $X=0$
~~HA*~~
SA* $y = X - 1$
crossing point(s)
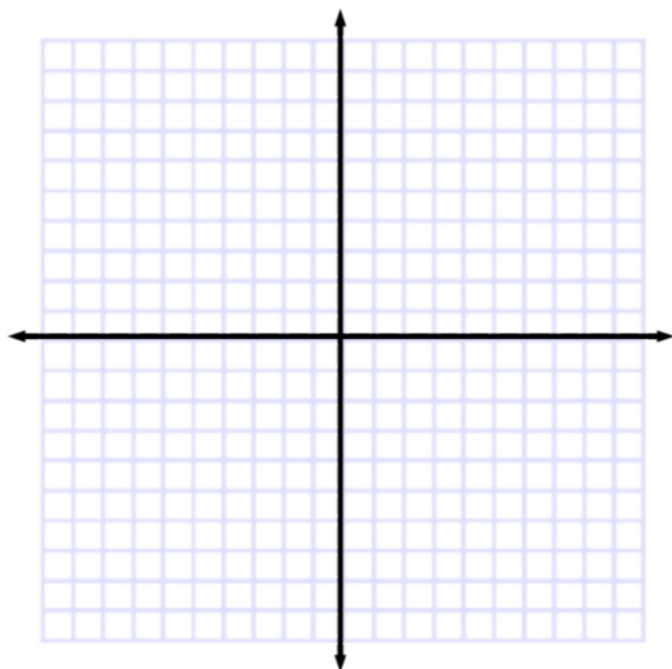Test point(s)
* Can have one or the other,
    but not both

$X \overline{\smash{\big)}\, X^2 - X - 2}$
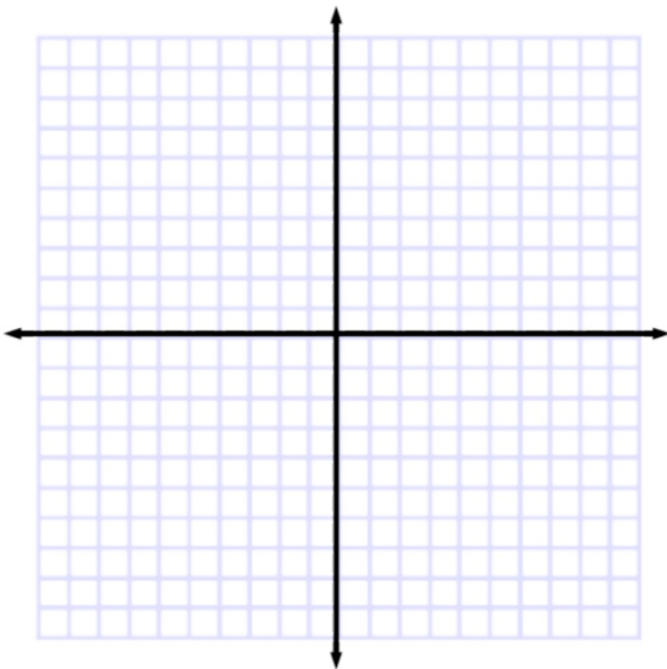
$-X^2$

$-X$

$+X$

-2

$15-33$ odds
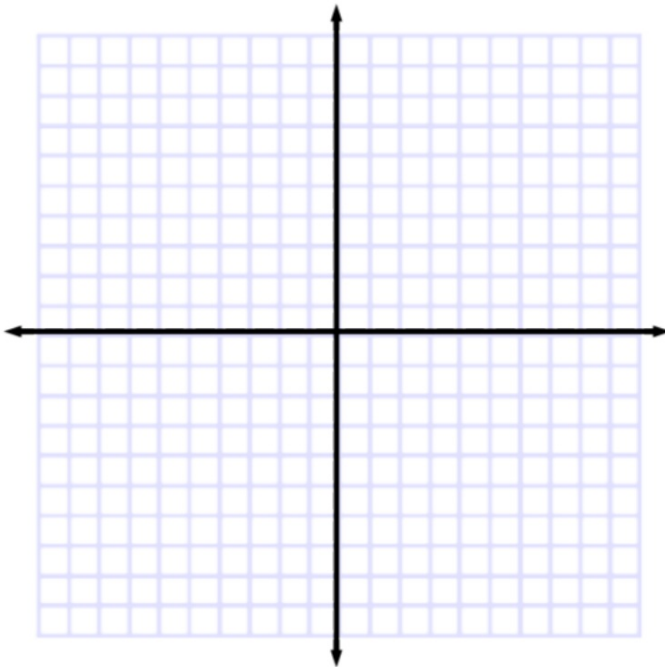
12. $y = \dfrac{x^2 + 4x + 4}{x + 2}$

Determine:
common factors
VA
HA
crossing point(s)
SA
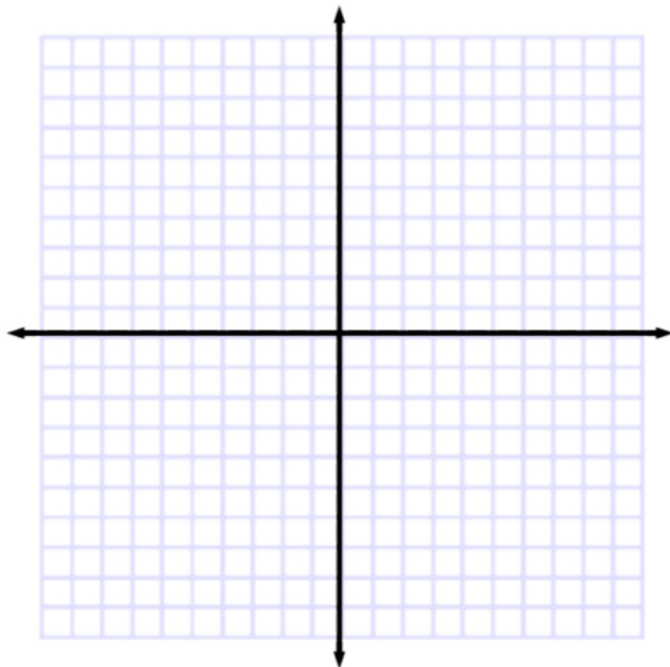Test point(s)

36. $y = \dfrac{x^2 - 4}{x - 2}$

Determine:
common factors
VA
HA
crossing point(s)
SA
Test point(s)

39. $y = \dfrac{x^2 - 6x + 9}{x^2 - x - 6}$

Determine:
common factors
VA
HA
crossing point(s)
SA
Test point(s)

5 Graph $y = \dfrac{(x + 3)(x + 1)}{x(x + 3)(x - 2)}$.

Determine:
common factors
VA
HA
crossing point(s)
SA
Test point(s)